CARL ZEISS VISION

**Memorandum**

TO: ANSI Z80 Subcommittee Members

CC:

RE: Developing Cylinder Axis Tolerances

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# Developing Cylinder Axis Tolerances

## Background

Over the past few years, the Optical Industry has been under fire for quality control issues. Often, sensationalistic journalists, with no understanding of our industry quality guidelines, have quite deliberately reached the erroneous conclusion that eyecare professionals are generally putting out poor quality work. Most commonly, the ANSI Z80.1 standard is misapplied as a "pass-fail" regulation, instead of as a set of voluntary guidelines, although the majority of the first section of the Z80.1 standard is devoted to clarifying this very point. One of the most problematic guidelines of the ANSI standards, and one of the more difficult to achieve by users of the standard, has been the tolerance on cylinder axis, particularly in low cylinder powers.

This memorandum will discuss how tolerances on cylinder axis are derived, and whether the current tolerances on axis for low cylinders powers are actually consistent with this methodology. Additionally, we will consider instances in which the tolerances on cylinder axis may not be realistic or practical. Since the current ANSI Z80.1–1999 standard is now under revision, we now have an opportunity to reevaluate these tolerances on cylinder axis, and to consider whether the industry may benefit from relaxing certain tolerances, if this can be accomplished without compromising visual performance for the wearer.

There are generally three criteria to consider when establishing optical tolerances:

1. **Process capability.** What can a typical process reasonably achieve?
2. **Measurement precision.** What can a typical instrument reliably measure?
3. **Visual significance.** What can a typical observer truly discern?

Process capability will be discussed when its consideration is relevant to the topic. For our purposes, it is important to realize that just because a given quality goal *can* be achieved at a certain cost does not necessarily mean that it is reasonable or necessary to do so. Conventional automatic and manual focimeters are capable of measuring deviations in cylinder axis to the precision that current cylinder tolerances demand, so measurement precision will not be addressed.[1] Consequently, this paper will focus on the visual significance of errors in the prescribed cylinder axis, as well as the optical justification behind establishing tolerances on cylinder axis.

## Understanding Errors in Cylinder Axis

Practically speaking, a spectacle lens of a given power is used to neutralize an ocular refractive error of equal magnitude but opposite sign. For instance, a spectacle lens with a power of −1.00 D is used to neutralize an ocular refractive error of +1.00 D (1.00 D of myopia) at the spectacle plane. Consequently, the effects of an error in the prescribed cylinder axis can be represented by combining two cylinder powers of equal magnitude but opposite sign at an angle equal to error in cylinder axis.[2] If we assume that their axes are not collinear—that is, the angle between the axes is between 0 and 90°—these two cylinders are said to be *obliquely crossed*.

Two obliquely crossed cylinders will result in a new sphero-cylindrical power, whose resultant sphere power, cylinder power, and cylinder axis will depend upon the magnitude of the original two cylinders and the angle between them. The resultant cylinder power $C$ produced by two individual cylinder powers $F_1$ and $F_2$, combined at an angle $\alpha$, is given by the following equations:

$$C = \frac{F_2 \cdot \sin 2\alpha}{\sin 2\theta} \qquad \textbf{Equation 1}$$

where $\theta$ is given by:

$$\tan 2\theta = \frac{F_2 \cdot \sin 2\alpha}{F_1 + F_2 \cdot \cos 2\alpha} \qquad \textbf{Equation 2}$$

Moreover, in our particular case—wherein the cylinder powers ($F_1$ and $F_2$) are equal in magnitude and opposite in sign—the equations can be simplified considerably. In this special case, the resultant cylinder power $C$ produced by combining two cylinders powers of equal magnitude ($F = F_1 = -F_2$), one negative and one positive, is given by:[3]

$$C = 2F \cdot \sin \alpha \qquad \textbf{Equation 3}$$

Equation 3 is also the equation for a *Stokes Lens*. This resultant cylinder power is essentially a residual refractive error produced by the misalignment of the cylinder axis of the spectacle lens with the prescribed axis. Furthermore, the *mean power* (or *spherical equivalent*) remains unchanged in this case, regardless of the original or resultant cylinder power. Consequently, only the resultant cylinder power is meaningful.

For instance, given a prescribed cylinder power of 2.00 D and an error in the prescribed cylinder axis of 5°, the resultant cylinder power error is:

$$C = 2(2.00) \cdot \sin 5 = 0.35$$

Consequently, an error from the prescribed axis of 5° produces a residual astigmatic (cylinder power) error of 0.35 D. Therefore, the wearer acceptance of a sphero-cylindrical lens with an error in cylinder axis can be considered in terms of its capacity to blur vision as a normal power error would. In this example, the error in axis is essentially equal to an unwanted cylinder power error of 0.35 D.

## Establishing Tolerances on Cylinder Axis

For the purposes of establishing prescription tolerances, we might ask how far the axis of a given cylinder power must be shifted in order to introduce an error in cylinder power equivalent to our normal (ANSI Z80.1) cylinder power tolerance. This would allow us to arrive at a reasonable baseline for establishing cylinder axis tolerances based upon the assumed visual significance of a comparable power tolerance, which has indeed been the historical approach to establishing cylinder axis tolerances. After rearranging the Equation 3 to solve for the angle $\alpha$, we have:

$$\alpha = \sin^{-1}\left(\frac{C}{2F}\right) \qquad \textbf{Equation 4}$$

Now, if we assume a tolerance in cylinder power of 0.13 D, which is the current Z80.1–1999 tolerance on cylinder powers up to 2.00 D, we can solve the angle $\alpha$ as a function of cylinder power, as shown in Figure 1.[4]

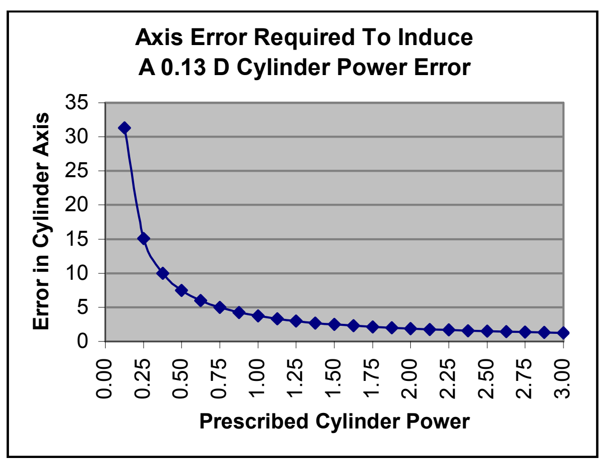


*Figure 1. Error in cylinder axis required to introduce a 0.13 D error in cylinder power*

For instance, we can see that a prescribed cylinder power of 1.00 D requires an error from its prescribed axis of nearly 4° in order to induce 0.13 D of unwanted cylinder power. It should be apparent from the figure that the results of the function become asymptotic for extreme cylinder powers. As the prescribed cylinder power approaches infinity, the error in axis required to induce the cylinder power tolerance approaches zero. Furthermore, as the prescribed cylinder power approaches its lowest limit, equal to one-half the cylinder power tolerance, the required error in cylinder axis approaches 90°.

Fry cited a similar methodology for the establishment of axis tolerances with the ANSI Z80.1–1979 standard.[5] Although Fry's results were derived from different equations, they are in fact equivalent to the results produced by the simpler formula given by Equation 4. Fry's results were based upon a cylinder power tolerance of 0.12 D, however, instead of the current tolerance of 0.13 D. For the sake of comparison, Table 1 shows the errors in cylinder axis necessary to induce both a 0.12 D error in cylinder power (used by Fry) and a 0.13 D error in cylinder power (the current ANSI Z80.1–1999 tolerance). The current ANSI Z80.1–1999 tolerances on cylinder axis have also been included.

*Table 1: Error in cylinder axis required to induce 0.12 and 0.13 D cylinder power errors*

| Cylinder Power | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | 2.25 | 2.50 | 2.75 | 3.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Axis Error @ 0.12 D** | **13.9°** | **6.9°** | **4.6°** | **3.4°** | **2.8°** | **2.3°** | **2.0°** | **1.7°** | **1.5°** | **1.4°** | **1.3°** | **1.1°** |
| **Axis Error @ 0.13 D** | **15.1°** | **7.5°** | **5.0°** | **3.7°** | **3.0°** | **2.5°** | **2.1°** | **1.9°** | **1.7°** | **1.5°** | **1.4°** | **1.2°** |
| **Z80.1 Tolerance** | **7.0°** | **5.0°** | **5.0°** | **3.0°** | **3.0°** | **3.0°** | **2.0°** | **2.0°** | **2.0°** | **2.0°** | **2.0°** | **2.0°** |

As previously stated, these tolerances were based in no small part on Fry's methodology. We can draw some interesting conclusions from this table. While the tolerance on cylinder power was loosened slightly in subsequent revisions of the Z80.1 standard, the tolerances on cylinder axis were *not* adjusted accordingly. Furthermore, the axis tolerances for the 0.25 and 0.50 D cylinder powers are *not* consistent with the results obtained using either the 0.12 D or the 0.13 D cylinder power tolerance in Equation 4. Note that the tolerance for 0.25 D of prescribed cylinder power, in particular, is considerably tighter than necessary.

Also note that the minimum tolerance is 2° for high cylinder powers, though the results actually demand slightly tighter values. In terms of mechanical alignment, it is generally assumed throughout the ANSI Z80.1–1999 standard that a tolerance of 2° is reasonable for errors in angular alignment (for example, segment tilt). Consequently, while there may be optical justification for a tighter tolerance in higher cylinder powers, the tolerance has been intentionally limited out of an assumption of reasonable process capabilities.

## Blur Produced by Unwanted Cylinder Power

There is a single location, lying at the dioptric midpoint of the two focal lines that bound Sturm's interval in an astigmatic focus, where the bundle of rays produces a circular cross-section. This location is referred to as the *circle of least confusion*. As stated earlier, the resultant cylinder power error produced by a misalignment in cylinder axis represents a purely astigmatic error (Figure 2). That is, the error from the mean sphere power—or spherical equivalent—remains equal to zero. In this case, the circle of least confusion of the astigmatic bundle will fall upon the retina, no matter how far the cylinder axis is off from the prescribed axis.

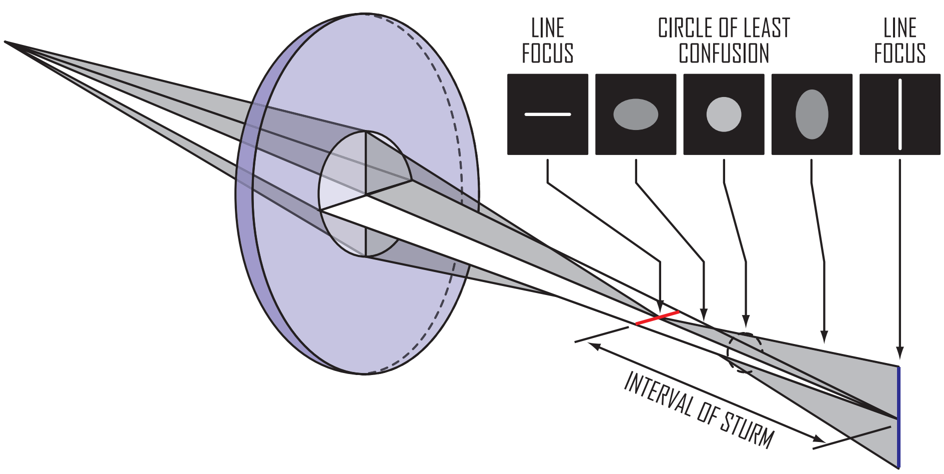


*Figure 2. The images of a point object resulting from the astigmatic focus produced by a lens with cylinder power contains two focal lines associated with each principal meridian and the circle of least confusion at the dioptric midpoint*

The circle of least confusion is proportional to both the pupil size and power error. As the size of the circle increases, so do the diffusion of visual information and the perception of blur by the wearer. Moreover, the circular patch of "blur" produced by the resultant cylinder power error at the circle of least confusion is *half* the size of the circular patch of blur produced by a spherical error of equal magnitude. Consequently, a given dioptric error in cylinder power produces half as much blur as an equal dioptric error in sphere power.[6]

For instance, recall that an error from the prescribed axis of 5° for a cylinder power of 2.00 D produces a residual astigmatic (cylinder power) error of 0.35 D. The blur produced by this cylinder error is comparable to the blur produced by an error in mean sphere power of 0.35 ÷ 2 = 0.175 D.

At this point, it might be reasonable to ask why the ANSI Z80.1–1999 tolerance on cylinder power is comparable to the tolerance on sphere power if its effects upon vision are less significant. There are several factors to consider. Firstly, power tolerances are generally based upon one-half the smallest increment of measurement used during ocular refraction. Cylinder power is generally prescribed in 0.25-diopter increments, so a tolerance of roughly 0.25 ÷ 2 = 0.125 D makes sense from a practical and metrological standpoint.

Secondly, the tolerance on sphere power does not actually apply to the mean sphere power (or spherical equivalent) of the prescription, but rather to the power through one principal meridian. For a cylinder power tolerance of ±0.13 D and a sphere power tolerance of ±0.12 D, the error in mean sphere power can reach up to 0.13 ÷ 2 + 0.12 = 0.185 D, or the sum of the sphere power tolerance and half the cylinder power tolerance. Consequently, higher cylinder power tolerances effectively increase the maximum sphere power tolerance.

Lastly, the additive nature of the tolerances on cylinder must also be considered, since errors in cylinder axis *and* cylinder power are both allowed. If the tolerance on cylinder *axis* allows up to 0.12 or 0.13 D of induced cylinder power, this will compound any error in the actual cylinder *power*, itself.

## Low Cylinder Powers and Progressive Addition Lenses

One of the most problematic aspects of cylinder tolerances is the control of cylinder axis in extremely low cylinder powers. In particular, progressive addition lenses, which often have small amounts of cylinder power as a result of the surface astigmatism on the progressive surface, are susceptible to this problem. Laboratory technicians often find it difficult to achieve the desired prescription in low cylinder powers when processing progressive lenses.

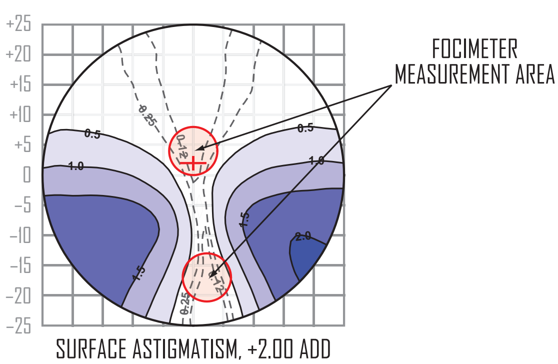


*Figure 3. Due to the highly aspheric nature of progressive surfaces and other manufacturing factors, the aperture of a conventional focimeter may "pick up" unwanted cylinder power in the vicinity of the power measurement points*

Due to the highly aspheric nature of progressive surfaces, the techniques used to manufacture the molds, and the variations produced during the manufacturing process, it is not unlikely for a progressive lens surface to have a small amount of unwanted surface astigmatism at the distance and near reference points, directly from the manufacturer. Progressive lens surfaces are highly aspheric and have a great deal of surface astigmatism, as a consequence of providing a progressive change in add power. This surface astigmatism results in unwanted cylinder power.

Even though the near and distance reference points of a progressive lens may produce no unwanted cylinder power in the initial lens design, manufacturing factors—such as lens material shrinkage—can create variations in the surface powers of the lens blank. Moreover, the apertures of many focimeters, which may be up to 8 mm in diameter, can "pick up" some of the unwanted cylinder power surrounding the distance and near reference points of the lens, resulting in erroneous power readings (Figure 3). Finally, some modern progressive lenses may have a small amount of cylinder power *intentionally engineered* into lens design at the distance and/or near measurement points in order to compensate for the effects of lens tilt and the *position of wear* on the power of the lens as perceived by the actual wearer. This is often referred to as *as-worn optimization*.

Because of the difficulties inherent in the production and measurement of progressive lenses, the International Organization for Standardization (ISO) has established looser surface power tolerances for progressive addition lens blanks (ISO 10322-2).[7] ISO tolerances allow 0.09 D of unwanted surface astigmatism in the distance zone of most progressive lens blanks, which is over two times what ISO tolerances allow for conventional bifocals. The unwanted cylinder power produced by this surface astigmatism interacts with any *prescribed* cylinder power on the back surface of the lens, resulting in a *crossed cylinder* effect, which can be computed using the mathematics described earlier (Equations 1 and 2).

Cylinder power on a lens surface, which has occurred either because of engineering, manufacturing, processing or measuring, will interact optically with any prescribed cylinder power. A consequence of this effect is a change in the axis of the prescribed cylinder power, particularly in weak cylinders. When one cylinder power is considerably stronger than the other, the resultant axis will shift very little from the axis of the stronger cylinder. Consequently, progressive lenses with a small amount of unwanted cylinder power in the distance zone will generally have very little effect upon stronger prescribed cylinder powers.

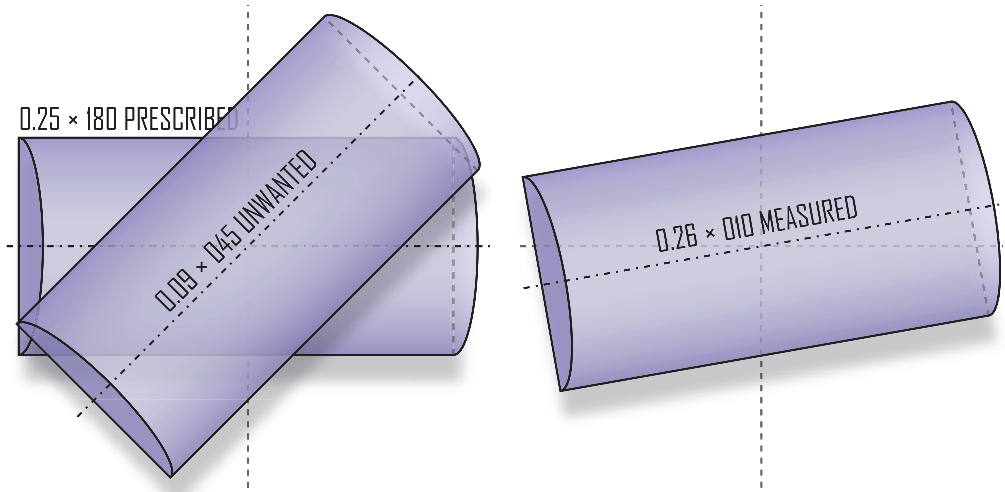


*Figure 4. Resultant or measured cylinder power produced by crossing the prescribed (back surface) cylinder with unwanted cylinder power on the front surface at a different axis*

However, when both cylinders are comparable to each other, the resultant axis will shift closer to the midpoint between them. Figure 4 demonstrates that combining a cylinder power of–0.09 D × 045, equal to the ISO tolerance for unwanted astigmatism on a progressive lens surface, with a *prescribed* cylinder power of –0.25 D × 180 results in a *new* cylinder power of –0.26 D × 010. This is 3° outside of the ANSI Z80.1–1999 tolerance. Consequently, it is possible for a precisely surfaced lens to meet the ISO standard while failing the recommended cylinder axis tolerance of the ANSI Z80.1 standard.

Fortunately, the small amount of unwanted cylinder power measured in the distance or near zone of a progressive lens is generally visually inconsequential to the wearer. Moreover, producing a progressive lens that *appears* to be virtually free from unwanted astigmatism at the distance reference point when measured across the range of focimeters currently in use would be both unnecessary and needlessly costly to progressive lens customers. In progressive lens designs with as-worn optimization, it may even represent a compromise in visual performance for the wearer.

## ANSI Compliance Study

In 1999, the OLA and VCA jointly sponsored a study to investigate the compliance to the ANSI Z80.1 standard by wholesale optical laboratories in order to determine whether the ANSI tolerances were realistic and representative of the current state-of-the-art of the industry. Roughly 800 prescription lenses (or 400 pairs) were evaluated and, of those, 600 contained prescribed cylinder power. These lenses were prescription "jobs" that were released by the laboratories to their customers after they were evaluated.

If the next revision of the ANSI Z80.1 standard returns to the use of the *sphere power* tolerance, as opposed to the use of the *meridian of highest power* tolerance, which is one of the last vestigial remains from the ISO-inspired ANSI Z80.1–1995 standard, the highest failure rate in meeting power tolerances for these 800 lenses occurs with cylinder axis. Applying the current ANSI Z80.1–1999 tolerances on cylinder axis to the 600 lenses with prescribed cylinder power results in a failure rate of approximately 9.3% (Figure 5).

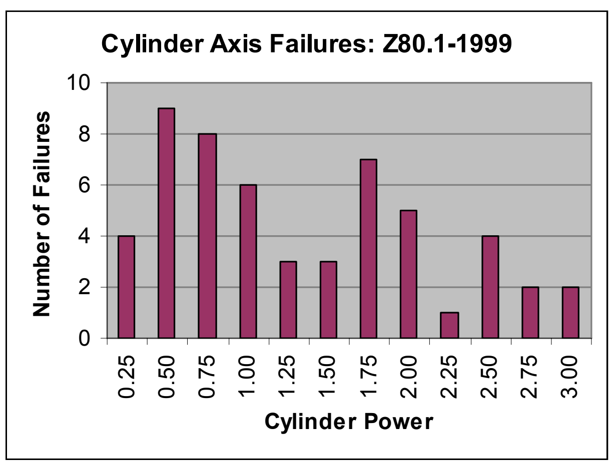


*Figure 5. Distribution of cylinder axis failures as a function of cylinder power using the ANSI Z80.1–1999 tolerances*

Now, what if we evaluate these data using more reasonable criteria for the lower cylinder powers? Choosing axis tolerances for the 0.25 and 0.50 D cylinder powers that were more in line with the methodology used to derive the tolerances for the other cylinder powers would improve our failure rate considerably, without introducing any more optical error than already allowed for the higher cylinder powers. For instance, if we were to increase the axis tolerance on cylinder powers of 0.25 D to a more reasonable 9° and the tolerance on cylinder powers of 0.50 D to a more reasonable 7°, the failure rate drops to roughly 7.7% (Figure 6). This represents an 18% reduction in failures.

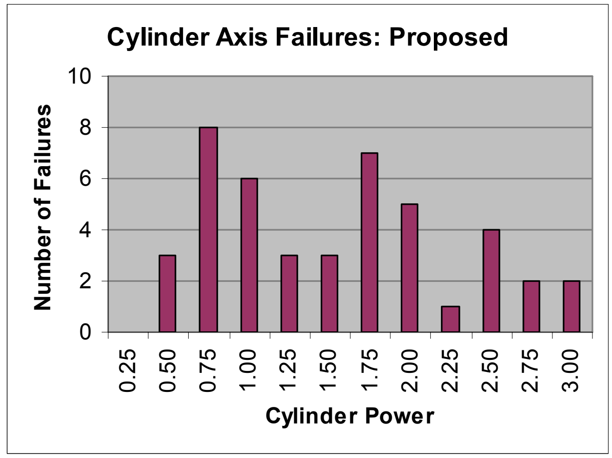


*Figure 6. Distribution of cylinder axis failures as a function of cylinder using the proposed tolerances*

Moreover, these slightly relaxed tolerances are quite reasonable and justifiable, and are actually *more* consistent with the optical criteria used to establish the other axis tolerances. Note that increasing the tolerance on cylinder axis for a prescribed cylinder power of 0.25 D even further, from 9° to something even more realistic—such as 11°, will result in only marginal improvements in the failure rate. In summary, we can literally "improve" the tolerances on cylinder axis by using a more consistent methodology while dramatically reducing industry reject rates. Consequently, it would behoove the Z80 Subcommittee to consider relaxing the tolerances on cylinder axis for the 0.25 and 0.50 D cylinder powers accordingly.

1 Fry, GA. "Tolerance for the Cylinder Axis." *Optical Index*. March 17, 1977. Pg. 20

2 Bennett, AG & Rabbetts, RB. *Clinical Visual Optics, 2nd Ed*. London: Butterworths, 1989. Pg. 87

3 Tunnacliffe, AH & Hirst, JG. *Optics*. London: Association of British Dispensing Opticians, 1996. Pg. 93

4 ANSI Z80.1–1999. *Prescription Ophthalmic Lenses – Recommendations*. American National Standards Institute

5 Fry, GA. Pg. 22

6 Bennett, AG & Rabbetts, RB. Pg. 85

7 ISO 10322-2: 1996. Ophthalmic optics – Semi-finished spectacle lens blanks – Part 2: *Specifications for progressive power lens blanks*. International Standards Organization