



Memorandum

TO: Vision Council Lens Technical Committee
CC:
RE: Understanding the Position of Wear

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Understanding the Position of Wear

The *position of wear* or *as-worn position* refers to the position and orientation of the fitted spectacle lens relative to the visual system of the wearer. Due to the optical effects associated with oblique refraction through a spectacle lens, the position of wear of the lens has important visual consequences for the wearer. When the line of sight is incident upon the lens at an angle to the optical axis of the lens, an optical aberration known as *oblique astigmatism* is produced. Oblique astigmatism results in unwanted sphere and cylinder power errors that are perceived by the wearer as deviations from the desired prescription. Oblique astigmatism is introduced when either the lens is tilted in the position of wear or the wearer views an object through the periphery of the lens. In either case, the line of sight forms an angle to the optical axis of the lens (Figure 1).

Over the past century, a great deal of research and development has been devoted to minimizing oblique astigmatism associated with the angle of view in order to provide eyeglass wearers with a wider field of clear vision. Less attention has been paid to the problem of oblique astigmatism associated with lens tilt in the position of wear. Conventional *best form* and *aspheric* lenses rely upon rotationally-symmetrical lens designs that can only fully correct the power errors associated with oblique astigmatism when the prescription calls for only negligible cylinder power and the position of wear of the fitted lens has only negligible tilt. Furthermore, due to the extensive computations required for eyecare professionals, prescription changes due to the position of wear, particularly those associated with lens tilt, have been largely ignored in the past.

Representing an improvement over the rotationally-symmetrical lens designs utilized for single-vision lenses, several modern progressive lens designs are now optically optimized for an average position of wear—assuming a spherical correction—in order to improve optical performance for many wearers. This involves fine-tuning the optics of the semi-finished lens design by modeling the performance of the eye-and-lens system using ray tracing. More recently, the convergence of improved eyewear measurement technologies with the advent of free-form lens surfacing has made possible the real-time optical optimization of spectacle lens designs for the exact position of wear and prescription requirements of each eyeglass wearer prior to fabrication. These enabling technologies allow free-form optical laboratories with sufficiently advanced lens design software to customize the optics of single-vision and progressive lens designs for the individual eyeglass wearer. Consequently, with the promise of improved visual performance for eyeglass wearers through the application of value-added design enhancements, there has recently been a great deal of interest in the optical principles and practical implications associated with the position of wear.

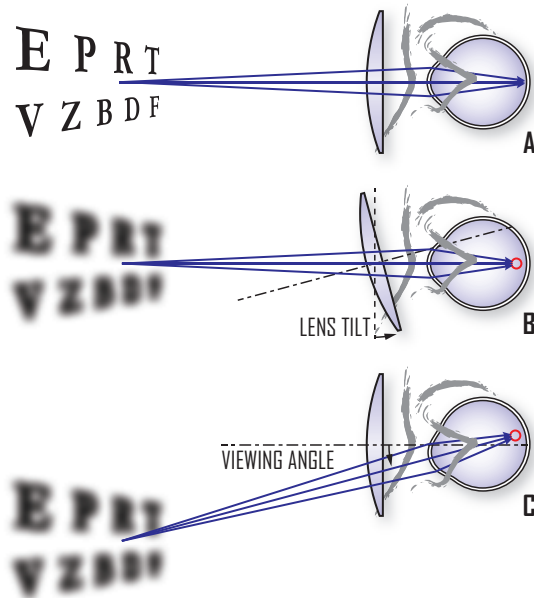


Figure 1. A) Oblique astigmatism is not produced when the line of sight is coincident with the optical axis of a spectacle lens; however, astigmatic power errors that the wearer perceives as a change from the desired sphere power and unwanted cylinder power occur when the line of sight forms an angle to the optical axis of the lens as a result of either B) tilting the lens in the position of wear or C) looking through the periphery of the lens at an off-axis viewing angle.

Defining the Position of Wear

Since spectacle lenses are held in place by an eyeglass frame or mounting, the orientation or “tilt” of a spectacle lens relative to the wearer is ultimately the net result of the tilt of the lens aperture of the fitted frame with respect to the wearer combined with the tilt of the finished lens with respect to the lens aperture that contains the lens. In practice, it is difficult to measure the tilt of a spectacle lens directly for several reasons:

- New eyeglass frames are glazed or mounted with temporary, non-prescription “demo” lenses, which will frequently not represent the geometry—that is, the thickness profile and surface curvatures—of the finished prescription lenses.
- Because most spectacle lenses are *meniscus* in form, having a convex front surface and concave back surface, factors such as lens decentration also influence lens tilt, since the optical axis is shifted along an arc roughly equal to the front surface (Figure 2).¹

$$\text{Tilt} = \arcsin\left(\frac{\text{Dec}}{R_1}\right)$$

- Due to the complex geometry of certain spectacle lenses, such as progressive lenses and lenses with prismatic components, the so-called “tilt” of the lens or of the optical axis may not be easily defined or readily measured.

Consequently, opticians typically measure the tilt of the eyeglass frame, not the tilt of the spectacle lens. For free-form lenses that are customized for the position of wear, many optical design systems therefore accept *frame tilt* values as input, which are then converted into *lens tilt* values relative to the wearer, once the geometry of the finished lens has been estimated.

After the orientation of the lens with respect to the wearer has been calculated, ray tracing can then be applied to evaluate an eye-and-lens model.

Since the human eye is in a constant state of motion, defining the orientation of a stationary spectacle lens with respect to the moving visual system of the wearer relies upon an assumption of a suitable reference position of gaze. The direction of gaze is represented by the line of sight that joins the object point of fixation to the center of rotation of the eye, which is also referred to as the *fixation axis*.² The eye is in *primary gaze* when looking straight ahead at a distant object with the head and shoulders erect.³ Spectacle lenses are generally fitted with the eyes in primary gaze.

The *plane of the frame front* is the plane containing the vertical midlines of the right and left boxed lens apertures. In the absence of frame tilt, the vertical and horizontal midlines of each boxed lens aperture are parallel to the plane of the frame front. Further, in the absence of tilt, each lens aperture is orthogonal—or perpendicular—both vertically and horizontally—to the line of sight of the eye in primary gaze (Figure 3). Customized lenses are typically fitted to pupil center while the wearer is looking straight ahead, so that the line of sight (or fixation axis) intersects the location of the fitting point of the lens when the eye is in primary gaze.*

The *position of wear* may be defined as the position and orientation of the fitted spectacle lens relative to the eye in primary gaze. The *orientation* of the spectacle lens is most consistently defined by the *orientation of the lens aperture of the frame* that contains the lens with respect to a frontal reference plane that is orthogonal to the line of sight in primary gaze. In the absence of pantoscopic tilt, the plane of the frame front—passing through the vertical midline of each boxed lens aperture—will coincide with this frontal reference plane. Further, if the horizontal and vertical midlines of the boxed lens aperture of the frame are treated as basis vectors, an equation of the plane of the lens aperture relative to this frontal reference plane can be readily deduced along with the slopes.** Moreover, the orientation of the tilted lens aperture with respect to the line of sight in primary gaze is also equivalent to the tilt of the normal vector that is given by the cross product of the two basis vectors associated with the horizontal and vertical midlines of the boxed lens aperture.

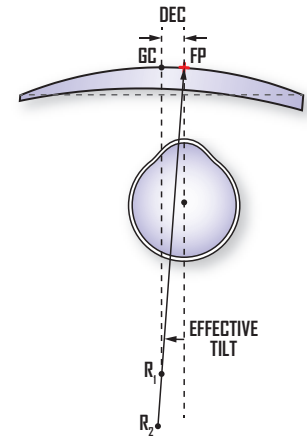


Figure 2. Decentration of a meniscus lens from the geometric center (GC) of the lens aperture to the fitting point (FP) typically occurs along an arc roughly equal to the curve of the front surface, causing the optical axis of the lens through the centers of surface curvature (R_1 and R_2) to assume an angle relative to the line of sight in primary gaze.

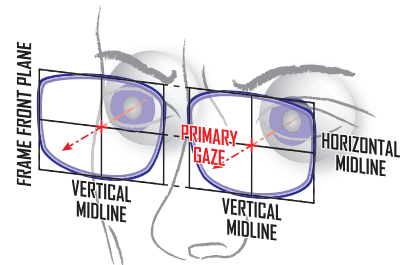


Figure 3. In the absence of frame tilt, the vertical and horizontal midlines of each lens aperture are parallel to the plane of the frame front and orthogonal to the line of sight in primary gaze; note that the fitting point has been positioned at the geometric center of the lens aperture in this diagram for simplicity, although the relationship between these planes and the line of sight holds, regardless of the location of the fitting point, during primary gaze.

* The lens designer may make a small compensation to account for the prismatic deflection of the line of sight at the fitting point of the lens.

** In reality, the horizontal and vertical midlines of the lens aperture may lie in two slightly different planes due to the curvature of the eyewire.

The *position* of the spectacle lens relative to the eye is typically defined by the *back vertex distance* of the lens, which is the longitudinal distance along the line of sight from the apex of the cornea to the back surface of the lens. The vertex distance may be defined with the line of sight perpendicular to the plane of the frame front or with the line of sight in primary gaze, depending upon the requirements of the lens designer. *The position of wear is therefore described by three fitting parameters associated with the tilt of the frame and the location of the lens* (Figure 4). Lens designers frequently use definitions for these parameters that are consistent with the ISO 13666 standard,⁴ a glossary of standardized terminology for spectacle lenses:

- *Pantoscopic tilt* represents the vertical angle between the plane of the frame front and a vertical plane orthogonal to the line of sight in primary gaze, which results from a rotation of the plane of the frame front around the horizontal X-axis.
- *Face-form tilt* represents the horizontal angle between the horizontal midline of the lens aperture and the plane of the frame front, which results from a rotation of the lens aperture around a vertical Y-axis in the plane of the frame front.
- *Back vertex distance* represents the longitudinal distance along the line of sight from the apex of the cornea to the back surface of the lens with the line of sight perpendicular to the plane of the frame front or, alternatively, in primary gaze.

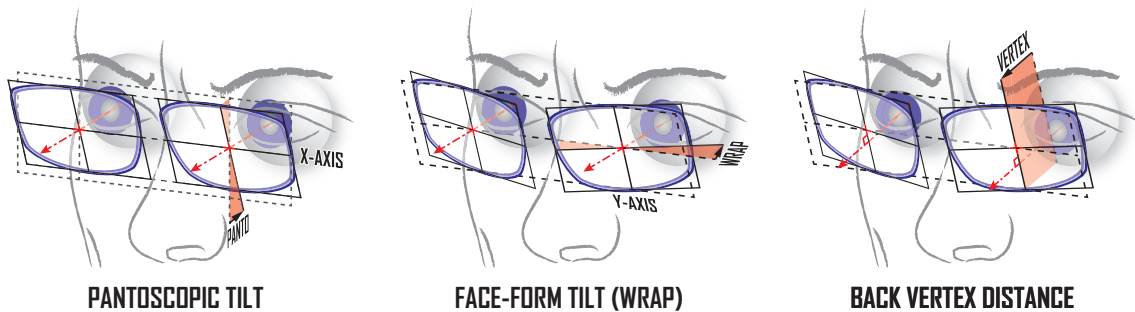


Figure 4. The orientation of the lens aperture is determined by the pantoscopic (vertical) tilt of the plane of the frame front around the horizontal X-axis and the face-form (horizontal) tilt of the lens aperture around a vertical Y-axis in the tilted plane of the frame front, while the position of the lens is determined by the back vertex distance along the line of sight from the apex of the cornea to the back surface of the lens with the line of sight perpendicular to the plane of the frame front or, alternatively, with the line of sight in primary gaze.

Positive *pantoscopic tilt* occurs when the bottom (inferior) edge of the lens aperture is tilted *toward the face*; *negative pantoscopic tilt*—or *retroscopic tilt*—occurs when the bottom edge is tilted *away from the face*. Positive *face-form tilt* occurs when the outer (temporal) edge of the lens aperture is tilted *toward the face*; *negative face-form tilt* occurs when the outer edge is tilted *away from the face*. Vertex distance is always positive. It is important to note that lens designers are ultimately interested in the *stop distance* to the center of rotation of the eye when ray tracing the eye-and-lens model, not the vertex distance. Consequently, some allowance must be made for the distance from the corneal apex to the center of rotation of the eye, unless this distance is also specified initially.

It may be appreciated that applying pantoscopic tilt to a frame that has face-form tilt effectively rotates the horizontal midline of the boxed lens aperture—and, therefore, the 180° meridian of the finished lens—slightly out of a horizontal plane, once the frame is in position. When a frame front is traced for lens edging, *pantoscopic tilt is eliminated by the mounting mechanism of the frame tracer*. Face-form tilt therefore becomes fixed with respect to the frame front as a rotation of the lens aperture around a vertical Y-axis in the plane of the frame front. When the wearer places the frame into position on his or her face, the *pantoscopic tilt as worn* effectively rotates the plane of the frame front around the horizontal X-axis, whereas the face-form tilt of the lens aperture and the orientation of the edged lens remain unchanged with respect to the tilted plane of the frame front.

Consequently, the horizontal rotation of face-form tilt effectively occurs after the vertical rotation of pantoscopic tilt in a sequence of gimbal-like *Euler rotations* that describes the final orientation of the plane of the lens aperture in three-dimensional space. Mathematically, a 3×3 Euler rotation matrix describes the linear transformation of basis vectors required to rotate the coordinate system of the plane of the lens aperture from the original frontal reference plane—orthogonal to the line of sight in primary gaze—to a new orientation with the specified combination of tilts. If the pantoscopic tilt angle P is first applied to rotate the plane of the frame front around the horizontal X-axis, followed by the face-form tilt angle F to rotate the lens aperture around a vertical Y-axis in the tilted plane of the frame front, this sequence of tilts yields the following Euler rotation matrix \mathbf{R} for the basis vectors associated with the vertical and horizontal midlines of the boxed lens aperture:⁵

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos P & -\sin P \\ 0 & \sin P & \cos P \end{bmatrix} \begin{bmatrix} \cos F & 0 & \sin F \\ 0 & 1 & 0 \\ -\sin F & 0 & \cos F \end{bmatrix} = \begin{bmatrix} \cos F & 0 & \sin F \\ \sin P \cdot \sin F & \cos P & -\sin P \cdot \cos F \\ -\cos P \cdot \sin F & \sin P & \cos P \cdot \cos F \end{bmatrix}$$

Measuring the Position of Wear

Eyecare professionals must rely upon practical measurements of the fitted frame that can be taken quickly and reliably in a clinical setting. The dispensing tools commonly available to eyecare professionals do not necessarily allow direct measurement of both the horizontal tilt and vertical tilt of the lens aperture of the frame relative to the frontal reference plane that was used to define the position of wear earlier. Consequently, the ISO 13666 standard essentially provides an *operational definition* of the position of wear by defining *three suitable clinical measurements that will allow a lens designer to establish mathematically the position and orientation of the finished lens in three-dimensional space*. There are now a variety of measurement technologies available for measuring the position of wear, ranging from simple protractors to sophisticated video centration devices, each with various advantages and disadvantages in terms of measurement accuracy, ease of use, equipment cost, and added functionality (Figure 5).

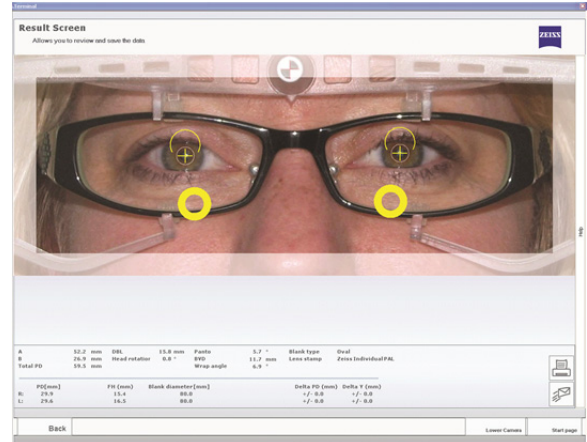


Figure 5. Computerized video centration devices are available to capture accurate measurements of the position of wear in addition to routine measurements of interpupillary distance and fitting height.

The position of wear of the fitted eyeglass frame can vary significantly from frame to frame and from wearer to wearer (Figure 6).⁶ Indeed, the range of possible measurement values associated with each fitting parameter underscores the potential importance of customizing the optics of the lens design for the position of wear. Of course, because frame adjustments will influence the position of wear, *the frame should be properly fitted to the wearer, prior to any measurements*.

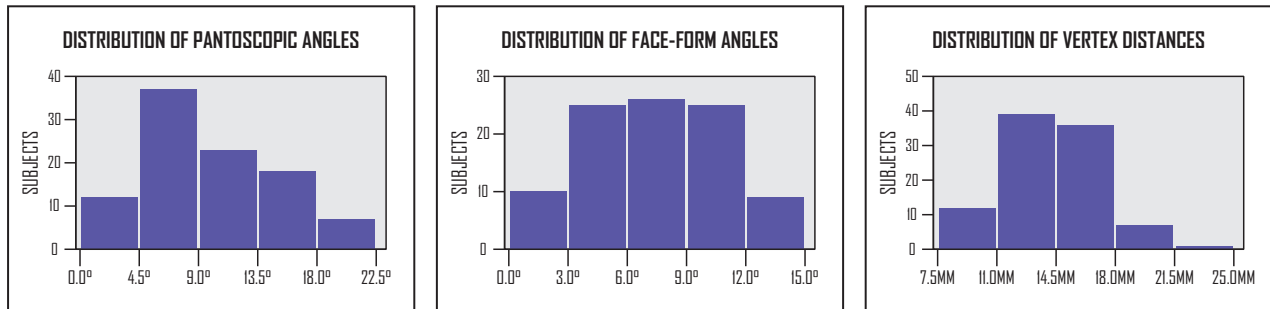


Figure 6. As measurements with a video central device from one study with 100 subjects demonstrate, the fitting parameters that define the position of the wear vary significantly among eyeglass wearers, emphasizing the potential benefits associated with optical customization for the position of wear.

Face-form tilt or “wrap” is often measured independently of the wearer using an inexpensive protractor (Figure 7). The horizontal angle formed between the horizontal midline of the lens aperture and the plane of the frame front is determined by measuring the angle that a line joining the nasal and temporal edges of the lens aperture across the midline makes with a plane parallel to the frame front. Alternatively, the angle between the two lens apertures can be measured, in which case the measured angle should be reduced by

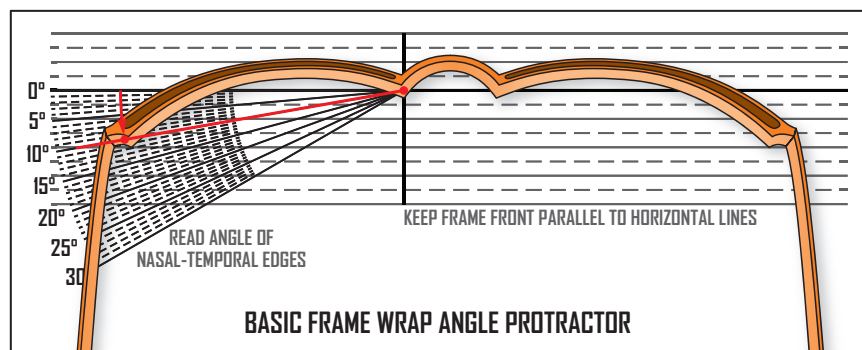


Figure 7. Face-form tilt can be determined using a simple “wrap protractor” to measure the angle formed between a line joining the nasal and temporal edges of the lens aperture and either a plane parallel to the frame front or another line joining the nasal and temporal edges of the opposite lens aperture, in which case the angle should be reduced by half; the lower edge of the frame front should be held directly over the upper edge in order to eliminate the influence of any pantoscopic tilt on the face-form measurement.

half. The lower edge of the frame front should be held directly over the upper edge to eliminate the influence of any pantoscopic tilt on the face-form measurement. The face-form tilt of the frame should also be verified on the wearer in order to ensure that the frame front does not exhibit unwanted “bowing” as a result of the normal forces exerted between the skull and the frame temples. Face-form tilt can also be captured by some video centration devices and “3D” electronic frame tracers.

Since it is assumed that the primary gaze direction is parallel to the ground, the force of gravity is frequently used to measure the pantoscopic tilt of the frame, as worn, by determining the vertical angle between the plane of the frame front and the frontal plane that is orthogonal to the line of sight. Using an inclinometer or similar tool that makes use of a plummet or spirit level, the vertical plane perpendicular to the ground is determined using gravity, while the tool is positioned against the frame front or the temporary lens, and aligned with the vertical midline of the boxed lens aperture. Note that *this angle will generally not be equal to the “pantoscopic tilt” of the plane of the frame front relative to the centerline of the temples*, as defined in the ANSI Z80.5 standard for frames,⁷ because *pantoscopic tilt as worn will vary with the fit of the frame on the actual wearer*. Video centration technology may rely upon various methods involving edge detection algorithms or the measurement of registration markers mounted to the frame in order to capture the pantoscopic tilt of the frame on the wearer.

The *measurement of as-worn pantoscopic tilt will be sensitive to the tilt of the head*, whether habitual or random. Although the plane that is orthogonal to the line of sight in primary gaze will often be parallel to the frontal plane of the skull, this may not necessarily be the case when the wearer has a habitual head tilt. Consequently, pantoscopic tilt is not necessarily associated with the plane of face, if the person has a *habitual* head tilt. However, if the person exhibits *unwanted* head tilt during the measurement, as the result of a random or atypical postural adjustment, this will result in an error in the measured angle of pantoscopic tilt (Figure 8). Therefore, when measuring pantoscopic tilt, the following requirements should be met:

- The wearer must be looking straight ahead, preferably fixating a distant object, in order to ensure that the line of sight remains in primary gaze, parallel to the ground.
- The fitting point of the lens must be aligned properly with the center of the wearer’s pupil to ensure that the line of sight intersects the fitting point of the lens during primary gaze.
- The wearer must assume his or her habitual posture, while minimizing any unwanted head tilt, in order to arrive at a truly representative measurement of the as-worn pantoscopic tilt angle.

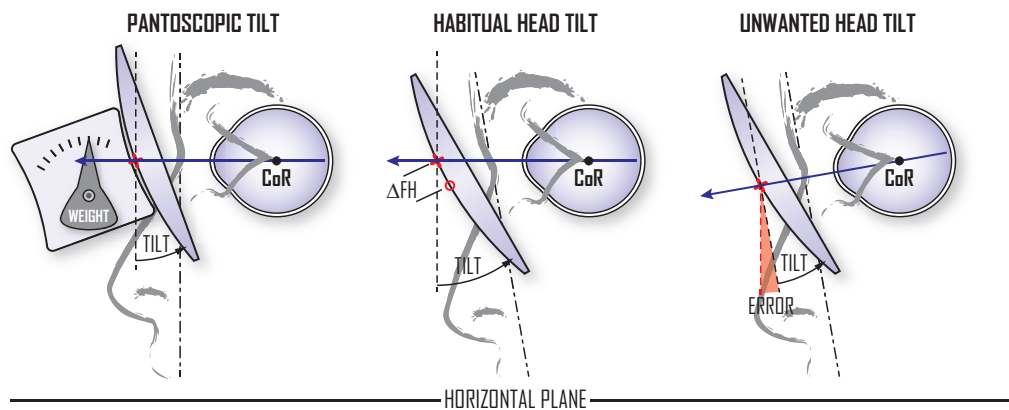


Figure 8. The pantoscopic tilt of the plane of the frame front is measured from a vertical plane orthogonal to the line of sight in primary gaze, often by positioning an inclinometer against the vertical midline of the boxed lens aperture; habitual head tilt will result in a valid change in pantoscopic tilt, although the fitting height will need to be modified accordingly to maintain the fitting point at the line of sight (by ΔFH); and unwanted head tilt will result in an error in the measurement of pantoscopic tilt.

Vertex distance is frequently measured manually using a ruler, pupillometer (held against the side of the face and eyewear), distometer, or other tool to determine the longitudinal distance along the line of sight from the apex of the cornea to the back surface of the temporary lens. When relying on the definition provided in ISO 13666, vertex distance is measured with the line of sight *perpendicular* to the plane of the frame front, which contains the vertical midlines of the right and left boxed lens apertures (Figure 9). This is generally also the shortest distance between the eye and the lens. Alternatively, the lens designer may define this measurement with the eye in primary gaze, so that the vertex distance is measured with the line of sight intersecting the intended location of the fitting point when the wearer is looking straight ahead. When in doubt regarding which measurement to use for a given lens design, the lens manufacturer should be consulted. Video centration technology can also capture measurements of vertex distance. Note that the vertex distance of the finished lens may differ slightly from the original measurement to the plane of the lens aperture or temporary lens, depending upon the prescription, form, and thickness of the finished prescription lens.

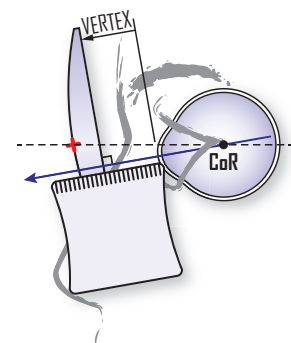


Figure 9. As defined in ISO 13666, vertex distance is measured from the corneal apex to the back of the lens, with the line of sight perpendicular to the plane of the frame front.

Understanding the Optical Effects of the Position of Wear

Spectacle corrections are typically determined using thin refractor-head or trial-frame lenses that are held perpendicular—or nearly so—to the line of sight and centered in front of the eye. *The line of sight is therefore coincident with the optical axis of the lens, precluding any optical effects associated with oblique refraction* (Figure 10). The refractionist determines the combination of sphere and cylinder powers that will neutralize the refractive error of the eye at the plane of the refractor-head or trial-frame lens, roughly 13.5 mm from the apex of the cornea. This represents the *refracted vertex distance*. The eyeglass wearer, on the other hand, experiences the optics of a spectacle lens with the lens mounted in a frame worn on the face in the *position of wear*. The orientation of the fitted spectacle lens will typically involve both pantoscopic and face-form tilt relative to the wearer. Additionally, the *fitted vertex distance* of the spectacle lens in the position of wear will often differ from the original refracted vertex distance.

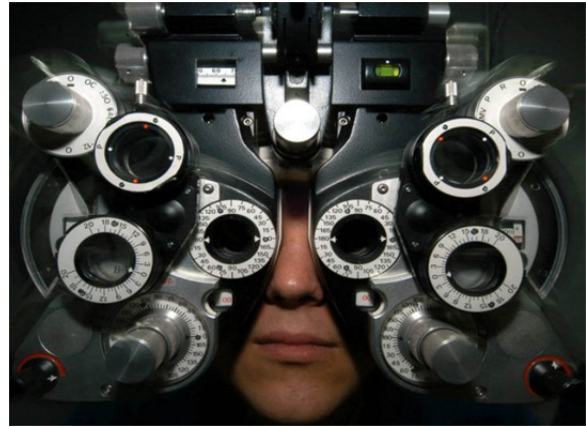


Figure 10. Spectacle corrections are determined using thin, flat refractor-head or trial-frame lenses that are typically positioned with the optical axis of the lens nearly coincident with the line of sight, precluding the effects of astigmatism due to oblique central refraction.

If there is a difference between the refracted and fitted vertex distances, the effective power of the fitted spectacle lens will be stronger or weaker relative to the prescribed power at the refracted vertex distance. The change in effective power $\Delta Power$ for straight-ahead vision will be approximately proportional to the difference in vertex distance and to the square of the power:

$$\Delta Power \approx \Delta Vertex \cdot Power^2$$

where the change in vertex distance $\Delta Vertex$ is expressed in meters. Vertex distance compensation is routinely applied to contact lenses to correct for the difference in power that results between the refracted vertex distance and the cornea. Moreover, the fitted vertex distance also affects the angles of incidence that the line of sight makes with the lens at different angles of gaze, which in turn influences the optical performance of the lens design during “off-axis” or peripheral vision.

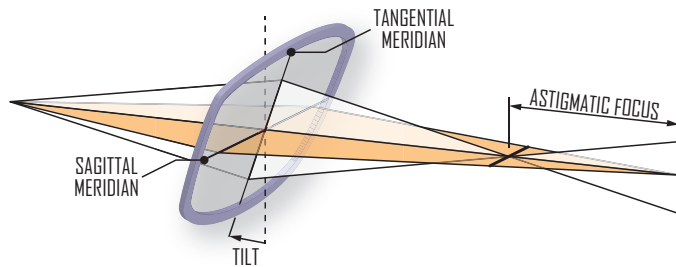


Figure 11. Tilting the optical axis of a spectacle lens relative to the intended image plane introduces a form of oblique astigmatism due to oblique central refraction, which increases the power through the tangential meridian of the lens, containing the plane of the angle of lens tilt, and through the sagittal meridian of the lens, perpendicular to it and containing the axis of lens rotation; however, the tangential power increases more rapidly than the sagittal power due to oblique refraction, resulting in an astigmatic focus, instead of a single point focus, with two focal lines crossed at right angles and separated by a distance equal to the astigmatic error.

Eyecare professionals are often less familiar with the effects of lens tilt on the specified prescription powers. *Tilting the optical axis of a lens introduces a form of oblique astigmatism, a lens aberration associated with oblique refraction.* Light rays refracted through the center of a tilted lens produce an astigmatic focus, with two focal lines separated by an interval of Sturm, because the power through the *tangential meridian* of the lens, containing the plane of the angle of lens tilt, increases more rapidly due to oblique refraction than the power through the *sagittal meridian*, containing the axis of lens rotation (Figure 11). For a tilted lens with spherical power, the sphere power of the lens increases and cylinder power (of the same sign) is introduced at an axis equal to the axis of rotation of the lens.

For a relatively thin lens with spherical power Sph_{RX} , the new sphere power Sph_{NEW} , and induced cylinder power Cyl_{NEW} due to oblique central refraction through a lens tilted by an angle θ can be calculated from Coddington’s equations:⁸

$$Sph_{NEW} = Sph_{RX} \left(1 + \frac{\sin^2 \theta}{2n} \right)$$

$$Cyl_{NEW} = Sph_{NEW} \cdot \tan^2 \theta$$

where n is the refractive index of the lens material. Plots of the change in sphere power and the induced cylinder power as a function of lens tilt for a range of spherical lens powers demonstrate that these power changes increase more rapidly with higher angles of lens tilt. Furthermore, the cylinder power increases more rapidly than the change in sphere power (Figure 12).

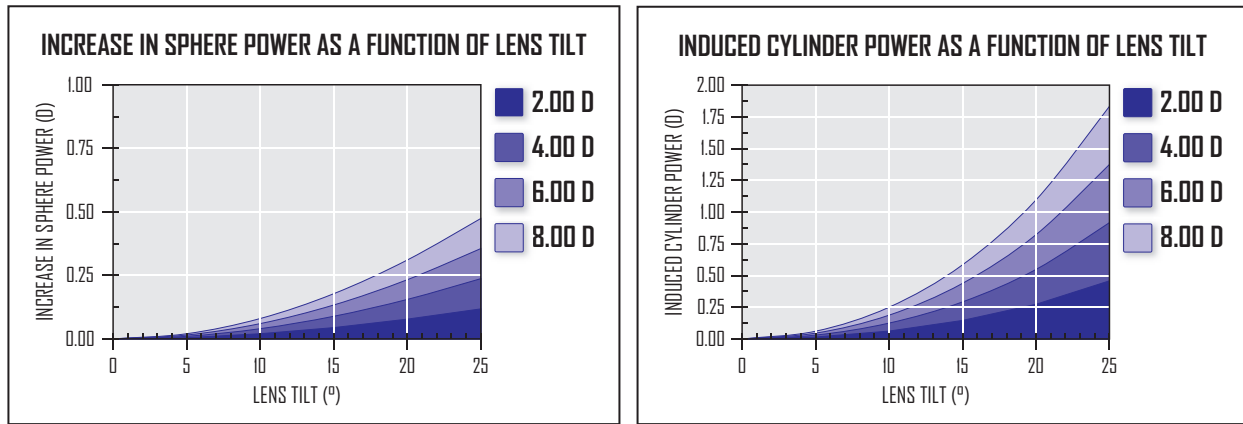


Figure 12. Plots of the increase in sphere power and the induced cylinder power as a function of lens tilt demonstrate that induced cylinder power increases more rapidly than the change in sphere power and that these power changes both increase more rapidly with increasing angles of lens tilt.

The axis of induced cylinder power is parallel to the axis of the lens about which the lens has been rotated; pantoscopic tilt around the horizontal axis of the lens produces cylinder power at axis 180, whereas face-form tilt around the vertical axis of the lens produces cylinder power at axis 90. The application of both pantoscopic and face-form tilt, simultaneously, results in an effective rotation of the lens around a new axis that is not collinear with either the vertical (90°) or the horizontal (180°) axis. In this case, the new axis of rotation and the resultant lens tilt produced by the combination of the two tilt angles can be calculated from an Euler rotation matrix. Additional computations are also necessary for lenses with prescribed cylinder power.⁹

It is possible to reduce the prescription changes introduced by lens tilt by modifying the sphere and/or cylinder powers of the original prescription in order to neutralize the oblique astigmatism produced by the tilted lens. It is also possible to reduce the oblique astigmatism introduced by pantoscopic lens tilt by lowering the optical center of the lens relative to the line of sight in primary gaze to maintain the intersection of the optical axis of the lens with the center of rotation of the eye (Figure 13). This is achieved by lowering the optical center by a distance ΔOC of roughly 1 mm for every 2 degrees of lens tilt θ .¹⁰

$$\Delta OC = 27 \cdot \tan \theta \approx \frac{1}{2}\theta$$

For this solution to work, *oblique astigmatism through the periphery of the lens must also be corrected using a best-form base curve, aspheric, or atoric lens design*, because the eye will effectively look through the lens “off-axis” during straight-ahead vision. This type of compensatory decentration of the optical center cannot be applied to face-form tilt, because horizontal prism imbalance will generally be induced. Moreover, excess vertical decentration can result in significant lateral chromatic aberration in stronger powers.

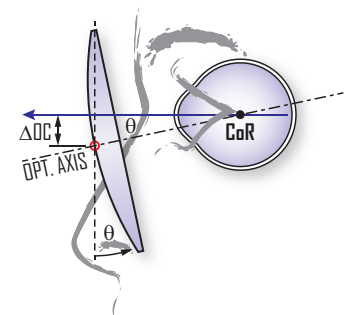


Figure 13. The oblique astigmatism produced by pantoscopic lens tilt can be reduced by lowering the optical center of the lens by roughly 1 mm for every 2 degrees of lens tilt, which maintains the intersection of the optical axis of the lens with the center of rotation of the eye.

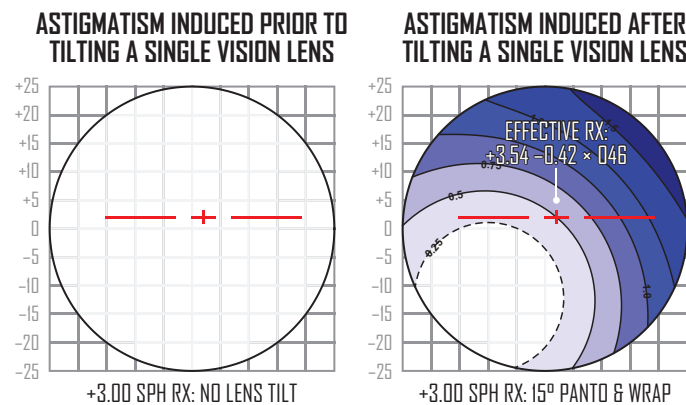


Figure 14. The oblique astigmatism introduced by tilting a spectacle lens results in a change from the desired prescription when looking straight ahead through the center of the lens as well as varying optical power errors over the entire lens aperture during peripheral vision away from the center of the lens.

It is important to emphasize the fact that *modifying the lens powers or decentring the optical center in order to compensate for the optical effects of the position of wear only improves vision straight ahead*. There are actually two separate optical consequences associated with the position of wear, a *static* optical effect and a *dynamic* optical effect (Figure 14):

- A change from the original prescription occurs when looking straight ahead due to oblique central refraction, resulting in a reduction in visual clarity through the center of the lens during sustained vision.
- Astigmatic power errors of varying magnitude occur when looking away from the center of the lens, resulting in varying levels of image degradation through the periphery of the lens during dynamic vision.

Compensating Prescriptions for the Position of Wear

If the orientation of the fitted lens is tilted in the position of wear, astigmatism due to oblique central refraction will result in a change from the desired sphere power and the introduction of unwanted cylinder power. Additionally, if the position of the fitted lens differs from the refracted vertex distance, a change in effective power due to vertex distance will also result in changes from the desired prescription powers. In order to provide the wearer with the intended prescription when actually wearing the spectacle lenses, these prescription changes must be essentially reversed from the original prescription. Neutralizing the optical effects of these prescription changes will require a new, compensated prescription to provide the wearer with the intended spectacle refraction during straight-ahead vision when the lenses are in the position of wear.

The compensated sphere power Sph_{COMP} and cylinder power Cyl_{COMP} required to counteract the power changes associated with the position of wear during straight-ahead vision for a thin lens of prescribed spherical power Sph_{RX} can be estimated with:

$$Sph_{COMP} = \left(\frac{Sph_{RX}}{1 + \Delta v \cdot Sph_{RX}} \right) \left(\frac{2n}{2n + \sin^2 \theta} \right)$$

$$Cyl_{COMP} = -Sph_{COMP} \cdot \sin^2 \theta$$

where n is the refractive index, θ is the angle of lens tilt, and Δv is the increase in fitted vertex distance from the refracted distance in meters (Δv is negative for a decrease in vertex distance). The compensated cylinder axis is equal to the axis of lens rotation—for example, axis 180 for pantoscopic tilt and axis 90 for face-form tilt—prior to any conversion for cylinder form. When the prescription calls for cylinder power or the frame has a combination of pantoscopic tilt and face-form tilt, additional computations are necessary to determine the compensated prescription (see Keating, 1995). It is important to note that a tilted lens with prescribed cylinder power will not necessarily have the same compensated cylinder axis. For thick lenses, lenses with prism, or progressive lenses, accurate ray tracing is generally necessary to determine the compensated prescription.

Consider, for example, a spectacle refraction of -6.00 D Sph at 13.5 mm that will be fitted at 9.5 mm with 12° of pantoscopic tilt: The estimated compensated prescription for a relatively thin lens in hard resin ($n = 1.500$) is -5.53 DS -0.25 DC $\times 090$.

Additional optical effects are associated with the near addition power of multifocal lenses in the position of wear. The near refraction to determine the addition power is conducted using trial lenses that are thin, flat, and centered. Spectacle lenses, on the other hand, have non-negligible thickness and curvature, resulting in potentially significant differences in optical vergence for near objects relative to trial lenses of the same back vertex power. This difference in vergence is referred to as the *near vision effectivity error*. Furthermore, because the near zone of a progressive lens is located at a significant distance below the center of the lens, the line of sight assumes a significant angle of ocular depression while reading. As a result of oblique astigmatism, optical power errors are often produced at this off-axis viewing angle during near vision that will also influence the effective addition power of the lens.¹¹ Due to these optical interactions, the increase in the back vertex addition power of a progressive lens will typically differ from the ray-traced addition power observed by the actual wearer with the lens in the position of wear (Figure 15). For free-form progressive lenses that have been optically customized for an assumed position of wear, the compensated addition power may therefore differ from the original near refraction value.

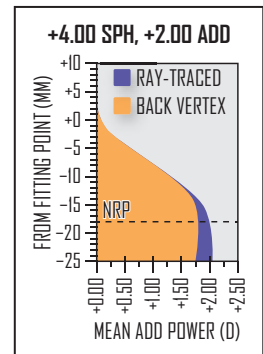


Figure 15. The back vertex add power measured by a focimeter will differ from the ray-traced power observed by the wearer.

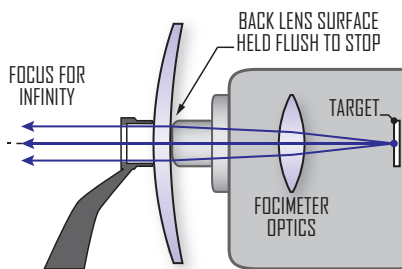


Figure 16. A focimeter measures the focal length of a spectacle lens with the lens centered and held flush against the lens stop of the instrument, which ensures that the optical axis of the lens coincides with the optical axis of the instrument, similar to the conditions of ocular refraction.

Spectacle corrections are determined using thin refractor-head or trial-frame lenses that are positioned in front of the eye so that the line of sight remains coincident with the optical axis of the trial lens. The prescription powers of spectacle lenses are typically verified using a *focimeter*, such as a lensometer or automatic lensmeter. Focimeters measure the focus of a spectacle lens while holding the center of the lens flush against the lens stop of the instrument, which ensures that the optical axis of the lens coincides with the optical axis of the instrument in the absence of prism (Figure 16).¹² A focimeter therefore closely replicates the measurement conditions associated with the original spectacle refraction using trial lenses, in which the optical axis is collinear with the line of sight. Hence, spectacle lenses have traditionally been designed and fabricated to provide the correct power readings for a focimeter, not necessarily the actual eyeglass wearer, under the assumption that the results would be comparable to the original refraction values.

When the powers of a spectacle lens have been compensated for the position of wear, however, legitimate differences should now be expected between the fabricated lens powers measured in a focimeter and the original spectacle refraction values. When evaluating optical quality against industry standards such as ANSI Z80.1 and ISO 8980-2,^{13,14} the powers of the spectacle lens should be verified against the compensated prescription when one has been supplied. *A spectacle lens that exhibits the correct compensated prescription and addition powers in a focimeter will provide the actual wearer with the correct prescribed powers once the lens is in the specified position of wear.*

With the introduction of free-form lenses, compensated prescriptions have become increasingly common. For the most advanced free-form progressive lenses that are optically optimized for the position of wear, the net optical effect produced by the combination of oblique astigmatism due to off-axis viewing and surface astigmatism due to the progressive optics is calculated at numerous points over the lens design by ray tracing the eye-and-lens model. Variable asphericity is then applied to each point over the lens design in order to provide the correct optical powers at every angle of view in the position of wear (Figure 17). Afterward, compensated prescription and addition powers are calculated at the distance and near reference points of the lens for power verification using a conventional focimeter. Note that, because the angle of depression required to reach the near zone of the lens may reach up to 30° or more, *the compensated addition power may differ significantly from the prescribed addition power.*

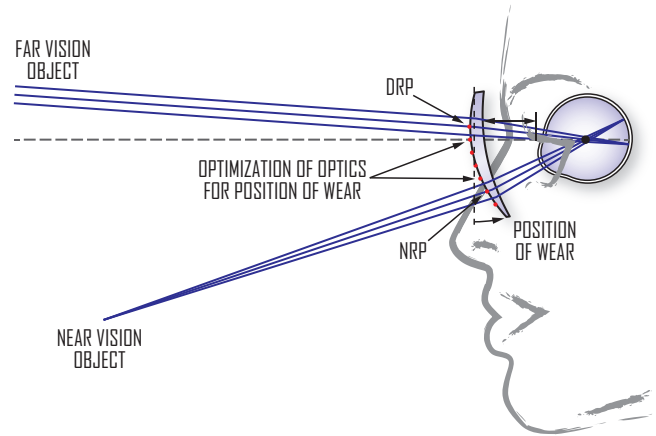


Figure 17. The most advanced progressive lenses customized for the position of wear are optically optimized using ray tracing by applying point-by-point asphericity over the entire lens design to provide the correct optical powers at every angle of view; afterward, compensated prescription powers at the distance and near reference points (DRP and NRP) are determined for the optical laboratory to supply with the lens for power verification using a focimeter.

Alternatively, it is also possible to customize the optics of the lens design for the position of wear, while constraining or offsetting the powers of the lens at the distance and near reference points in order to preclude the use of a compensated prescription. This approach results in lenses that can be verified directly against the original spectacle refraction values, for convenience, at the expense of some reduction in optical performance in the vicinity of these measurement points. Furthermore, if specific position-of-wear values are not supplied or utilized, *a default position of wear that represents reasonable population averages may be assumed* during the optical optimization process used to design the lenses.

It should be emphasized that compensating the prescription, alone, will generally not provide the wearer with optimum optical performance. While prescription compensation will reduce the power errors through the center of the lens, the optical performance away from the center will still suffer from oblique astigmatism due to lens tilt and to off-axis viewing angles. Consequently, *while modifying the specified prescription powers to account for oblique central refraction can improve visual acuity when looking straight ahead, providing the wearer with an unrestricted field of clear vision over the entire lens can only be achieved by optically optimizing the lens design for the anticipated position of wear* (Figure 18).

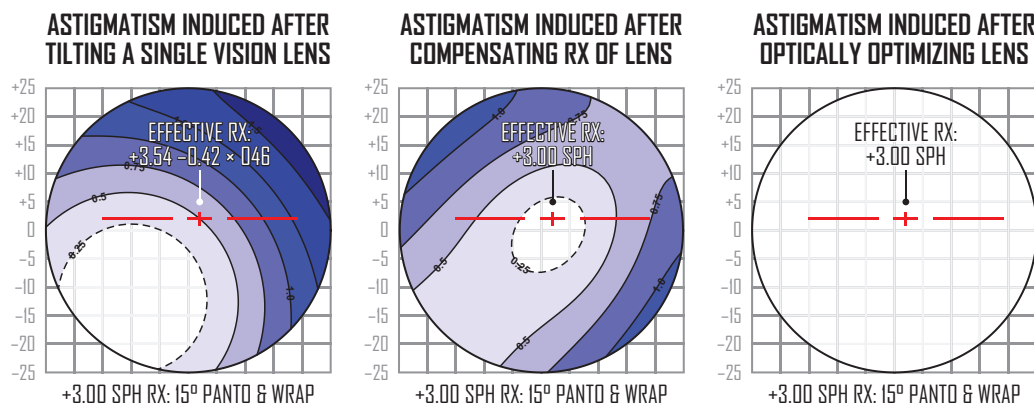


Figure 18. Significant lens tilt introduces astigmatic power errors over the entire lens; however, compensating the prescription for the position of wear primarily improves straight-ahead vision by correcting for astigmatism due to oblique central refraction through the center of the lens, whereas optically optimizing the lens design for the anticipated position of wear by applying point-by-point asphericity can virtually eliminate oblique astigmatism over the entire lens.

Customizing Optics for the Position of Wear

Conventional spectacle lenses are produced from *semi-finished* lens blanks that are factory-molded in mass quantity. Due to the product development and inventory costs associated with semi-finished progressive lenses, in particular, each lens design is typically available in only a handful of base curves. The lens design associated with each base curve must therefore work sufficiently well for a relatively broad range of possible combinations of prescription powers and fitting parameters. Single-vision lenses are subject to similar limitations. Most commonly, conventional lenses are designed to work optimally for a handful of spherical prescription powers—one for each base curve—in an average position of wear. For semi-finished progressive lenses, this is often accomplished by fine-tuning the optics of the lens design using ray tracing to model the optical performance of the lens in an average position of wear using the median sphere power of the prescription range associated with each base curve.

Consequently, while some wearers may enjoy the intended optical performance with conventional lenses, *many wearers must tolerate reduced optical performance as their prescription or fitting requirements depart from the assumptions used to design the semi-finished lens blanks from which their lenses were made.* These optical compromises are particularly problematic for progressive lenses, since the prescription and position of wear can introduce oblique astigmatism that interacts optically with the surface astigmatism of the lens design, thereby causing the viewing zones to lose clarity and to shift, shrink, or become distorted in shape. The wearer therefore experiences a reduction in the utility of the viewing zones of the lens due to power errors as well as a disruption in binocular fusion due to differences in power and prism between the right and left lenses.

Fortunately, the advent of “free-form” technology has freed many lens designers from the constraints of traditional mass lens production by enabling an optical laboratory to deliver spectacle lenses that have been designed and manufactured in “real time,” after the actual lens order has been placed. *Free-form surfacing* is a manufacturing platform that allows the production of a virtually unlimited variety of smooth optical surfaces, on demand, in a small-scale production environment.¹⁵ In addition to conventional toric surfaces, free-form surfacing allows an optical laboratory to produce progressive, aspheric, and other complex lens surfaces. When used in conjunction with sufficiently advanced optical design software, known as a *lens design server*, free-form surfacing can be utilized to deliver customized *single-vision and progressive lens designs that have been fully parameterized using prescription and fitting input specific to the individual wearer, immediately prior to fabrication* (Figure 19).^{16,17}

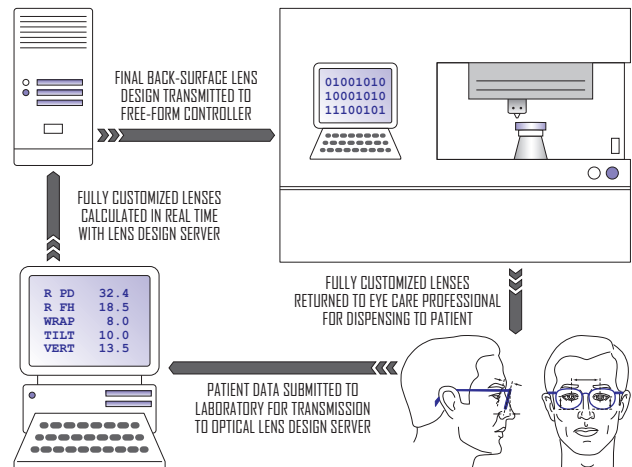


Figure 19. The optical design of the most advanced free-form lenses is optically customized in real time by a lens design server, using prescription and fitting data specific to the individual wearer, before the final lens surface is transmitted to free-form surfacing equipment for fabrication.

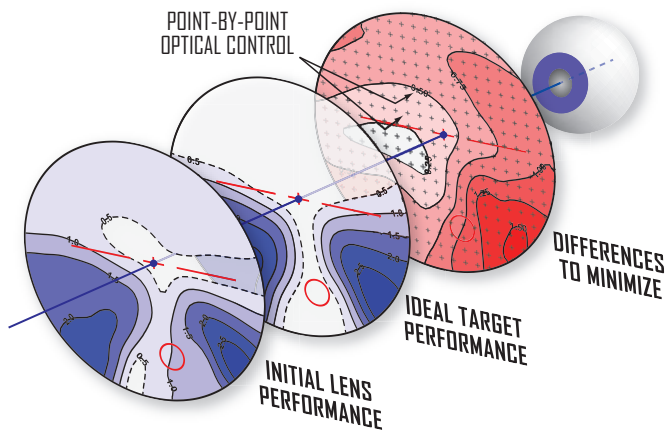


Figure 20. For the more advanced free-form lenses that are customized for the wearer, numerical optimization methods are utilized to find a lens surface that minimizes as much as mathematically possible the ray-traced optical differences between the initial optical performance of the lens in the actual position of wear and the ideal performance of the lens at hundreds of points over the lens design.

Numerical optimization methods are frequently utilized to fine-tune the optical performance of a customized lens by minimizing as much as possible the ray-traced optical differences between the initial performance of the lens, determined for the specified prescription and fitting parameters, and the ideal performance of the lens for a median sphere power in a reasonable position of wear (Figure 20). In the application of this technology by one free-form lens manufacturer, an initial (starting) surface is first calculated for the lens that provides the specified prescription powers. A mathematical model of the eye and the spectacle lens with the specified pantoscopic tilt, face-form tilt, and vertex distance is then ray traced. The ray-traced optical performance of the lens is compared against a target distribution of optical quantities that represents the ideal distribution of ray-traced optical characteristics such as mean add power and unwanted astigmatism at predefined points over the lens design.¹⁸

The curvatures at hundreds of points over the initial lens surface, smoothly joined using *bivariate splines*, are then manipulated in order to minimize the optical differences from the target distribution at specified points over the lens. This point-by-point manipulation results in a complex aspherization of the lens surface that varies continuously over the lens design. Generally, a smooth surface cannot achieve the desired target distribution, at least for every point. Numerical optimization methods therefore seek to minimize the differences between the desired optical performance over the lens and the actual optical performance possible with a continuously smooth surface (Figure 21). This is often accomplished by minimizing a *merit function* (M) of optical metrics at each specified point, which is a cost function used to find least-squares solutions of the form:¹⁹

$$M = \sum_{i=1} w_i \cdot (A_i - T_i)^2$$

where for each optical metric i assessed, A_i is the actual value of the metric, T_i is the target value, and w_i is the weighting factor assigned to the metric at a given point. The merit function values for the specified points are integrated over the entire lens. The most common optical metrics to minimize are mean power error (defocus) and unwanted astigmatism. The weighting factors may vary as a function of location in order to emphasize different performance attributes over various regions of the lens; metrics associated with clear vision, for instance, are often more heavily weighted at points within the central viewing zones.

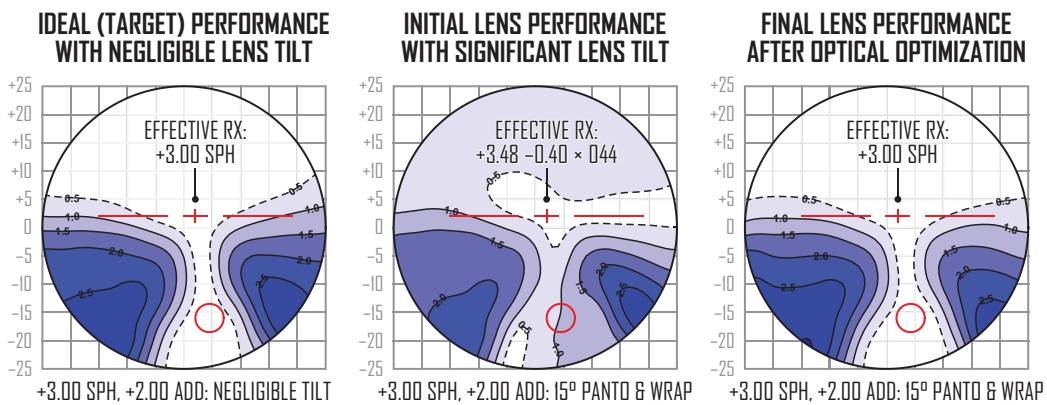


Figure 21. Free-form lenses from Carl Zeiss Vision that are customized for the position of wear rely on numerical optimization methods to minimize as much as mathematically possible the differences between the ideal (or “target”) optical performance and the actual optical performance of the lens when modeled with the specific prescription and fitting parameters of the wearer.

Conventional progressive lenses can suffer from significant degradations in optical performance for those wearers whose prescription or fitting requirements differ significantly from the parameters originally assumed to design the semi-finished lens blank. Free-form lenses that are optically customized for the position of wear using numerical optimization techniques, on the other hand, will preserve the intended optical performance of the lens design, regardless of the specific prescription or fitting requirements of the wearer (Figure 22). Consequently, *all wearers should enjoy consistently good optical performance with free-form lenses that are customized for the position of wear, at least within the optical limits of the target lens design.* It is important to note, however, that *not all free-form technologies apply optical customization for the wearer.*²⁰

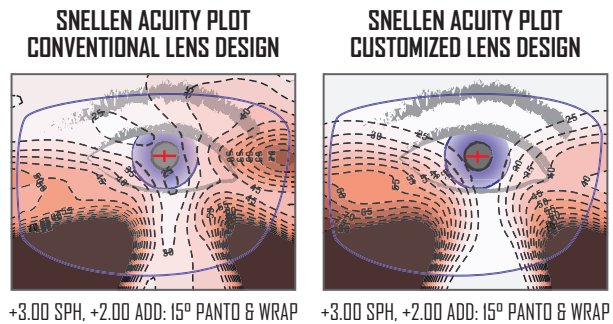


Figure 22. Although conventional progressive lens designs (left) can suffer from significant degradations in optical performance for many wearers, free-form lenses that are optically customized for the position of wear, such as those offered by Carl Zeiss Vision (right), will preserve the intended optical performance of the lens design for all wearers.

The visual benefits of customized progressive lenses from one lens manufacturer were recently confirmed by researchers at the School of Optometry at the University of California at Berkeley.²¹ The optical performance and wearer satisfaction of customized progressive lenses produced by Carl Zeiss Vision were compared against a variety of traditional, semi-finished progressive lenses. A total of 95 subjects completed a randomized, double-blind wearer trial that assessed objective measures of visual performance as well as subjective measures of wearer preference. At the time of dispense, wearers demonstrated greater high-contrast visual acuity on average with the customized lenses compared to traditional lenses. After one week of wear, 65% of wearers demonstrated a wider field of near vision with the customized lenses. Overall, *there was a clear preference for the customized lenses* after accounting for interaction effects using multivariate analysis ($p < 0.001$).

Improving Optics for Wrap Eyewear

Prescription sunwear is frequently worn during activities that require sharp vision over a wide field of view as well as minimal prism and distortion, which could otherwise impair comfortable binocular and dynamic vision. The position of wear associated with modern sunwear, however, often compromises visual performance and comfort with conventional lens designs. Many of these frames are steeply curved and highly “wrapped” with significant face-form tilt. Further, the spectacle lenses fabricated for these frames often rely on the use of non-optimal front curves in the 7- to 8-diopter range to better match the curve of the frame eyewire. *These steeply curved and highly tilted lenses disrupt optical performance for the wearer* in a number of ways:

- The extreme horizontal lens tilt caused by frames with significant wrap results in reduced visual clarity through the central region of the lens due to astigmatic power errors from oblique central refraction. These power errors cause an increase in sphere power and induced cylinder power, proportional to the power of the lens and to the degree of tilt.
- The introduction of optical aberrations in the periphery of the lens caused by substituting base (front) curves in order to match steep, “8 Base” frames results in a restricted field of clear vision due to oblique astigmatism. Spherical and aspheric lens designs cannot correct for oblique astigmatism in steeply curved and highly tilted lenses in most prescriptions.
- Horizontal lens tilt results in an apparent narrowing of the fabricated distance between the right and left lenses, related to the cosine of the wrap angle. The decentration of the fitting point and inset of the progressive corridor of each lens must therefore be modified in order to properly align the center of each viewing zone with the line of sight during fixation.
- The prismatic displacement of light rays caused by the front surface of steeply curved and highly tilted lenses results in horizontal prism imbalance that can impair comfortable binocular vision. When a spectacle lens is tilted by an angle θ , an unwanted prismatic effect Δ due to the oblique incidence of light rays with the front surface F_1 of the lens is given by:²²

$$\Delta \approx 100 \cdot \tan \theta \cdot \frac{t}{n} F_1$$

Progressive lenses are particularly sensitive to the optical effects associated with steeply curved and highly tilted lenses due to the presence of the surface astigmatism that is used to “blend” the distance and near zones of the progressive surface together. Oblique astigmatism exacerbates the surface astigmatism of the progressive lens design, which reduces the width of the progressive viewing zones, further restricting the fields of clear vision. Furthermore, optical interactions occur between the oblique astigmatism and the surface astigmatism, causing the viewing zones of the lens to shift and become distorted in shape, as certain regions of the lens become clearer than necessary while other regions become more blurred than intended. These misshapen viewing zones disrupt comfortable binocular fusion and reduce the binocular field of clear vision for the wearer.

Although many optical laboratories now offer “prescription compensation” to account somewhat for the prescription changes that occur due to lens tilt in highly wrapped frames, prescription compensation only improves vision quality through the central region of the distance zone. The optical performance over the rest of the lens is still compromised by residual optical aberrations due to lens tilt and to the use of non-optimal base curves. Astigmatic power errors over the rest of the lens can be *reduced by using an atoric or non-rotationally symmetrical lens design*. In order to *eliminate astigmatic errors over the entire lens*, however, point-by-point asphericity must be applied to one of the lens surfaces by *optically optimizing the lens design*, prior to free-form fabrication, using the specified prescription and the wrap angle of the chosen frame (Figure 23).

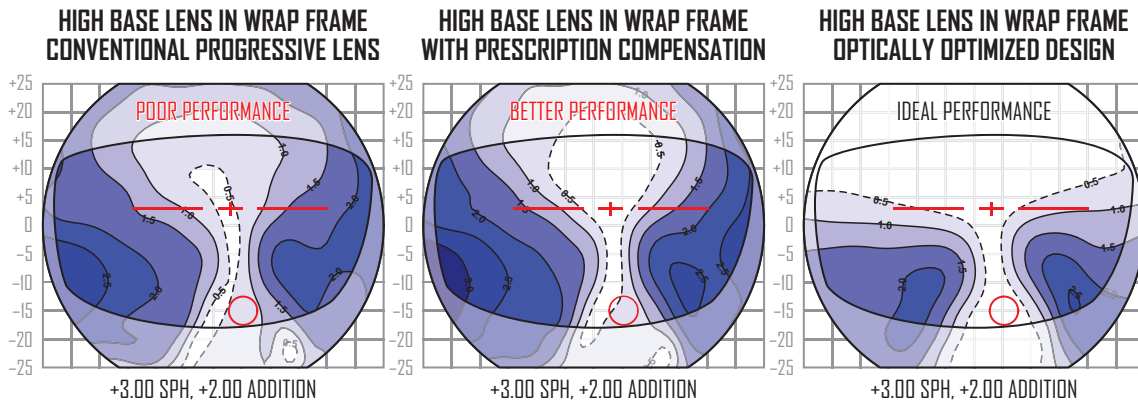


Figure 23. Steeply curved and highly tilted lenses for “wrap” eyewear introduce astigmatic power errors over the entire lens that can disrupt optical performance, particularly in progressive lenses; although compensating the prescription for the position of wear improves straight-ahead vision through the center of the lens by correcting for astigmatism due to oblique central refraction, the lens design must be optically optimized for the position of wear with point-by-point asphericity in order to eliminate oblique astigmatism over the entire lens.

Determining the Distance to the Center of Rotation

The movements of the eye as the wearer changes gaze resemble those of a ball-and-socket joint with a *center of rotation* located near the center of curvature of the posterior globe of the eyeball.²³ As the eye rotates behind a spectacle lens to fixate objects in the periphery, the peripheral regions of the lens should focus the pencil of light rays that passes through the pupil of the eye onto the fovea. The *chief* or *principal ray* of light passing through the center of the pupil will only intersect the fovea if the ray passes through the center of rotation of the eye as the eye rotates to different angles of view. The *aperture stop* of the eye is therefore positioned at the center of rotation when calculating the optical performance of the lens. Hence, the distance from the back vertex of the spectacle lens to the center of rotation of the eye is referred to as the *stop distance*.²⁴ Although the back vertex distance of the lens is commonly measured, *lens designers are ultimately interested in the total stop distance*.

When designing a spectacle lens, the stop distance to the center of rotation must be utilized in ray tracing procedures to calculate the angles of incidence and refraction associated with the line of sight as the eye rotates to different angles of gaze behind the lens (Figure 24). The optical power errors produced by oblique astigmatism vary as a function of the stop distance of the eye-and-lens system. Additionally, the calculations of the size of the viewing zones and the inset of the corridor of a progressive lens design also depend upon the total stop distance.

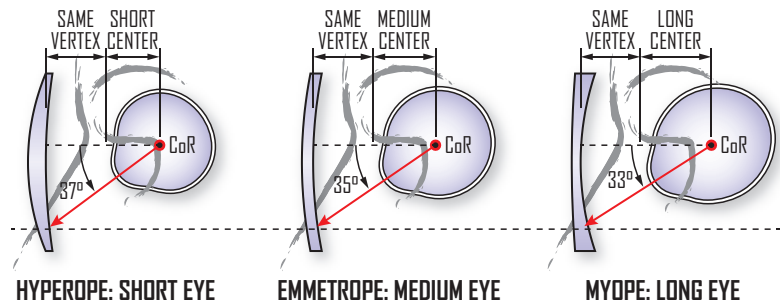


Figure 24. Because the length of the human eyeball varies due to factors such as refractive error, the distance from the cornea to the center of rotation of the eye (CoR) also varies, causing the line of sight to form differing angles of incidence with the lens at the same linear distance from the fitting point as the eye rotates, which ideally should be taken into account during ray tracing in order to achieve the desired optical performance and distribution of progressive optics.

The stop distance of the lens, which is equal to the sum of the back vertex distance from the lens to the cornea and the ocular distance from the cornea to the center of rotation, must be either assumed or physically measured, prior to lens design. Significant differences between the location of the center of rotation used for design calculations and the actual center of the wearer's eyeball can increase the unwanted astigmatism in the periphery of a progressive lens, decrease the utility of the central viewing zones, and reduce the binocular alignment between the right and left viewing zones when the eyes converge to read.

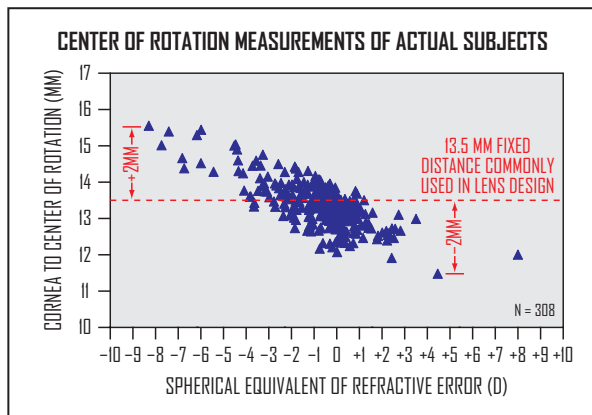


Figure 25. Historically, lens designers have assumed a fixed distance to the center of rotation, although anatomical measurements of actual human eyes using optical biometry vary considerably with refractive error.

The location of the center of rotation varies with the anatomical length of the eye, which in turn usually varies with the refractive error of the eye.²⁵ Optical biometry measurements of human eyes have confirmed this correlation (Figure 25). Hyperopic eyes are typically shorter than emmetropic eyes, while myopic eyes are typically longer. Although the importance of the center of rotation and the potential variation due to refractive error have long been recognized, a constant distance to the center rotation has often been utilized when designing spectacle lenses, regardless of the intended prescription.²⁶ Historically, a distance of 13.5 mm from the cornea to the center of rotation has been assumed, which is based upon a schematic emmetropic eye. However, some modern free-form lens designs now rely upon the distance to the center of rotation of the actual wearer when optically optimizing the lens design, prior to fabrication.

The back vertex distance is typically measured using a distometer, ruler, or centration device. For lens designs that are also customized for the ocular distance to the center of rotation of the eye, the distance from the cornea to the center of rotation is either mathematically estimated based upon the prescription of the wearer or, alternatively, physically measured by the eyecare professional using a special instrument. Although similar differences in either distance contribute equally to changes in the stop distance, there is significantly more variance in the back vertex distance of the lens—roughly 5 to 20 mm—compared to the center of rotation distance—roughly 11½ to 16 mm—among adult eyeglass wearers. Consequently, variations in the back vertex distance often contribute more to differences in the stop distance and, therefore, to optical performance. Nevertheless, *the center of rotation distance is an important lens design parameter associated with the position of wear*.

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