Clinical and Experimental Optometry: Authors' Version of the Work

Progress in the spectacle correction of presbyopia.
Part 1: Design and development of progressive lenses

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Most of the commercial advances in the spectacle correction of presbyopia continue to occur in progressive lens design, which has been the focus of intense research and development over the past sixty years by major spectacle lens manufacturers. While progressive lens design and manufacturing techniques have advanced at a steady pace, recent progress in "free-form" lens surfacing has opened up many exciting possibilities that will in all likelihood bring about a paradigm shift in the current model of progressive lens fabrication and distribution. The first installment of this two-part series will review the fundamental optical principles and early development work associated with progressive lenses.

Key words: lens design, presbyopia, progressive lenses, spectacle correction

Introduction

Conventional (that is, "lined") bifocal lenses offer two zones of fixed-focus vision, separated by a visible discontinuity or "ledge." In many cases, this discontinuity produces an abrupt change in image size and location, known as *image jump*, as the line of sight passes into the "segment" region. A band of blur and a potential blind area, or *scotoma*, in the visual field are produced as well, as the pupil is simultaneously exposed to two different power and prismatic effects while the line of sight passes over this discontinuity. Additionally, mid-range utility through bifocal lenses is often limited, particularly as the wearer's presbyopia advances.

Progressive lenses, on the other hand, are multifocal lenses employing a class of surfaces that provide a continuously smooth increase in positive focal power in order to compensate for accommodative insufficiency. Most commonly, the curvature of these surfaces gradually increases from a minimum value within the stabilized—or nearly spherical—*distance zone* on the front surface of the lens to a maximum value within the stabilized *near zone*, thus providing the desired change in near addition (or *add power*). Moreover, this gradual increase in curvature produces a *corridor* of progressively increasing plus power, effectively providing a variable-focus *intermediate zone*. These three zones are flanked to either side by "blending" regions of blur and geometric distortion (Figure 1).

Progressive lenses provide the desired addition power without any lines or ledges by essentially "blending" the transition between the distance and near zones. This blending is achieved by incorporating various amounts of

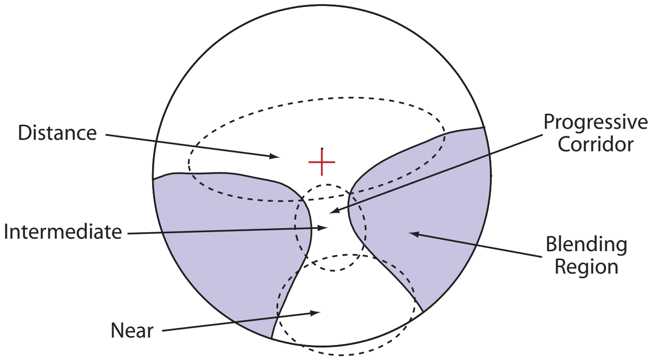

Figure 1. The structural features of a "general-purpose" progressive lens include zones of stabilized distance vision, stabilized near vision, and progressively increasing intermediate vision, with "blending" regions of unwanted blur and distortion to either side.

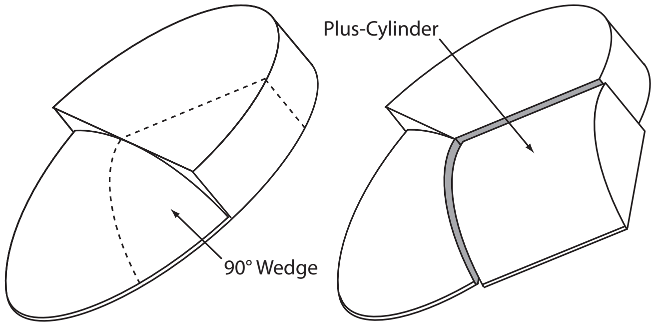

Figure 2. The ledge at the junction between a flatter curve and a steeper curve can be eliminated using cylinder power, as demonstrated by removing a 90-degree wedge from an Executive-style bifocal and replacing it with a section of a plus-cylinder.

surface astigmatism or *cylinder*, generally oriented at an oblique axis, in the lateral regions of the lens surface. The use of a plus-cylinder at an oblique axis to seamlessly join sections of two surfaces with different curvatures can be appreciated with the aid of Figure 2.[1]

Progressive lens surfaces are often described as "continuously smooth" surfaces that are "locally toric." A surface free of ledges and other physical discontinuities must have a *continuous surface height*. A surface that is physically smooth, with no sharp peaks or valleys and no abrupt changes in prism, must also have a *continuous first derivative* (that is, *surface slope*). Finally, a surface that provides only smooth, continuous changes in power and magnification must have a *continuous second derivative* (that is, *surface curvature*) as well. Such surfaces are sometimes called "C2" surfaces to reflect this mathematical constraint.

The optical and cosmetic advantages of progressive lenses are well known: Progressive lenses provide a continuous range of focus from near to far without any visible lines of demarcation, which would otherwise result in visually disturbing changes in image size and location. Arguably, progressive lenses replicate natural, pre-presbyopic vision more effectively than conventional bifocal lenses by providing a continuous depth of field with no abrupt changes in vision. Of course, the primary disadvantage to progressive lenses is the blur and geometric distortion produced within the so-called "blending" regions of the progressive surface. For several decades now, managing this blur and distortion has been a principal concern of progressive lens designers.

Early Progressive Lens Design

The optical principles of simple progressive-powered lenses have long been understood. The earliest progressive lens patent was submitted in 1907 by Owen Aves, co-founder of the London Refraction Hospital (now known as the Institute of Optometry).[2] His invention was a dual-surface progressive lens design that employed a section of a cone on one side and a section of an elliptic cylinder on the other, as illustrated in Figure 3. The cone provided a progressive increase in curvature through the horizontal meridians of the lens, while the elliptic cylinder provided a progressive increase in curvature through the vertical meridians roughly equal to the horizontal curvatures at corresponding points on the opposite surface.

Unfortunately, the lack of rotational symmetry and the dual-surface nature of the Aves design made it impractical for mass production as a prescription sphero-cylindrical lens, so it was never introduced commercially. Shortly thereafter, Henry Orford Gowlland invented a single-surface progressive lens design that employed a section of a paraboloid on the back surface.[3] Other progressive lens designs followed over the years,[4,5] although the marginal performance characteristics of these early lens designs, combined with the manufacturing challenges associated with the machining techniques available at the time, relegated this form of multifocal correction to little more than a novelty.

In fact, because of these limitations, progressive lenses failed to enjoy any real commercial success until the 1960s. Before the advent of computer-numerically-controlled grinding techniques, the mass production of complex progressive lens surfaces that lacked the symmetry of a

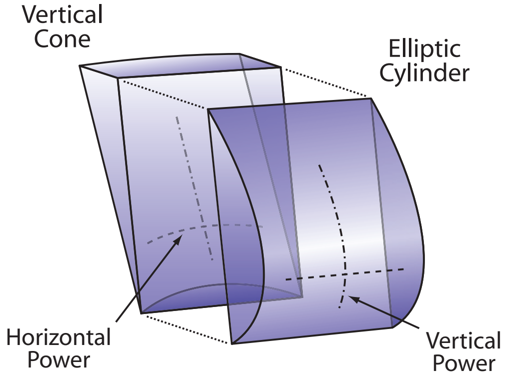

Figure 3. Owen Aves's original progressive lens concept incorporated a section of a cone on one surface in order to achieve a progressive increase in power through the horizontal cross-sections and a section of an elliptic cylinder on the other surface in order to achieve a progressive increase in power through the vertical cross-sections.

surface of revolution frequently met with insurmountable challenges. Novel manufacturing techniques were often devised in an attempt to fabricate these lens surfaces on an actual production basis and to make them a viable alternative to conventional bifocals lenses.

The development of the first commercially successful progressive lens, for instance, arguably represented a greater technical achievement in fabricating asymmetrical lens surfaces than in optical design. Mechanical cams were utilized to control the angle of contact between the surface of a lens blank and a standard grinding wheel as the profile of the surface was generated, as illustrated in Figure 4.[6] The evolute of the radii of curvature that produced the progression of addition power down the surface was controlled by the shape of one or more cams. The optical performance of this simple progressive lens, including the size of the central viewing zones, was largely dictated by the progression of addition power along the progressive corridor, which in turn was defined by the shape of these cams.[7]

Several early progressive lens designs, including Aves's original design, realized on a single surface, employed a class of surfaces similar in optical effect to the surface geometry of a curled elephant's trunk (Figure 5). Like an inverted cone, the progressive region of these surfaces could be represented by circular cross-sections that gradually decrease in diameter, thereby increasing in curvature, down the length of this theoretical "trunk." Further, this trunk is bent so as to ensure that the vertical curvature of the trunk matched the horizontal curvature at any point along the front of the trunk.[8]

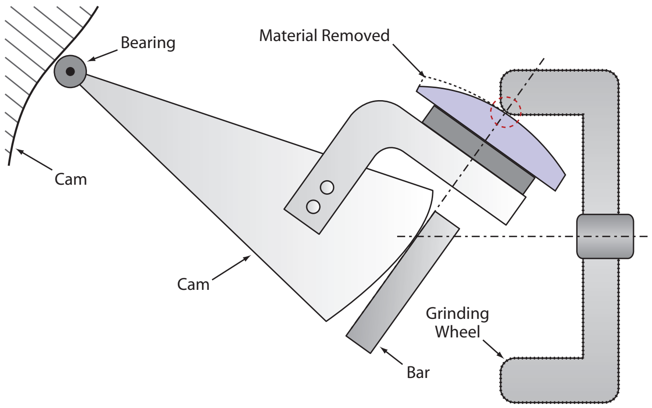

Figure 4. The progression of addition power of this early progressive lens was produced by controlling the angle of contact between the lens surface and a standard grinding wheel using one or more mechanical cams (modified from Cretin-Maitenaz, 1959).

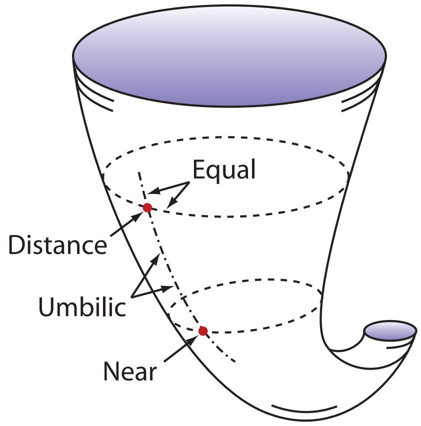

Figure 5. The net optical effect of several early progressive lenses, including the original dual-surface design of Owen Aves and the first commercially successful progressive lens, was similar in principle to an elephant-trunk-shaped surface (modified from Bennett, 1973).

The Progressive Lens Problem

Along the vertical centerline of the "elephant trunk" surface, the instantaneous curvatures at any small point are equal in every direction. Consequently, there exists a single vertical meridian that is essentially "spherical" at any point, which is referred to as the *umbilic* of the surface.* This meridian defines the centerline of the progressive corridor. Away from the umbilic, however, the minimum and maximum curvatures of the lens surface begin to depart, resulting in *surface astigmatism*. This surface astigmatism increases laterally into the periphery of the lens, resulting in significant quantities of unwanted cylinder power.

Much insight into the nature of progressive lens optics may be gained from an analysis of this simple lens surface. The rate of change in addition power along the umbilic of this surface is often referred to as the *power law* of the lens design. Although the definition differs in practice, the *corridor length* of the lens design can be defined as the vertical distance separating the minimum curvature within the distance zone and the maximum curvature within the near zone of the lens surface along the umbilic. From this, the *average* power law δ*Add* along the umbilic, in diopters per millimeter, can be determined from:

$$\delta Add = \frac{Addition}{Corridor}$$

…*Power law* [1]

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* Some modern progressive lens surfaces are actually designed with a small amount of cylinder power along the "umbilic."

If a constant power law with a linear increase in addition power is assumed for this surface, a fairly simple and well-documented mathematical model may be derived for the elephant trunk progressive surface. Although progressive lens designs generally employ a power law that varies non-linearly along the umbilic in order to provide stabilized zones of distance and near vision, this mathematical model is nevertheless useful for deducing some fundamental principles of progressive lens surfaces. Expressing this linear power law in terms of surface curvature, in units of reciprocal millimeters, yields:

$$g = \frac{\delta Add}{1000 \cdot (n-1)} \qquad [2]$$

where *g* is the rate of change in surface curvature along the umbilic. This power law is integrated once with respect to *y* in order to arrive at an equation of the surface curvature κ as a function of vertical position *y*:

$$\kappa(y) = g \cdot y \qquad [3]$$

For simplicity, the constants of integration will be left at zero. Integrating this equation a second time with respect to *y* to arrive at the slope of the surface and a third time to arrive at the height *z* of the surface as a function of *y* then yields:

$$z(y) = \tfrac{1}{6}\kappa(y) \cdot y^2 = \tfrac{1}{6} g \cdot y^3 \qquad [4]$$

which is the equation for the height of the surface along the umbilic. Since the horizontal cross-sections of the elephant trunk surface are essentially circular, these sections can be approximated by parabolas of the form ax^2, where 2*a* is equal to the curvature, when the surface is relatively flat. For the elephant trunk surface, the horizontal curvature 2*a* is equal to the vertical curvature κ at any point *y* along the umbilic, so that the height *z* of the surface as a function of *x*, at any vertical location *y*, is given by:

$$z(x) = a \cdot x^2 = \tfrac{1}{2}\kappa(y) \cdot x^2 = \tfrac{1}{2} g \cdot y \cdot x^2 \qquad [5]$$

Therefore, the final function for the entire surface is approximately represented by a third-degree polynomial of the form:

$$z(x,y) = \tfrac{1}{6} g \cdot y^3 + \tfrac{1}{2} g \cdot y \cdot x^2 \qquad [6]$$

which can also be expressed as:

$$z(x,y) = \frac{g}{6}\left(y^3 + 3yx^2\right) \qquad \text{…\textit{Elephant trunk surface}} \ [7]$$

Unless the surface is quite steep, the horizontal and vertical curvatures of this simple elephant trunk surface, which utilizes a linear power law, remain roughly equal into the periphery. Maximum surface astigmatism occurs, however, through the *oblique* meridians of the lens at axis 45 degrees. This surface astigmatism increases linearly away from the umbilic, producing significant quantities of unwanted cylinder power at axis 45 degrees. *Minwitz's theorem* states that the unwanted cylinder power *lateral* to the umbilic of this type of progressive surface increases twice as rapidly as the addition power increases *along* the umbilic, so that:[9]

$$\delta Cyl = 2 \cdot \delta Add \qquad \text{…\textit{Minkwitz's theorem}} \ [8]$$

where δ*Cyl* is the rate of change in cylinder power (or astigmatism) and δ*Add* is the rate of change in addition power (that is, the power law). For the simple elephant trunk surface, Minkwitz's theorem is just a consequence of the relationship between the third partial derivatives of the surface. The astigmatism, or difference in curvature, at axis 45 degrees (a_{45}) of this relatively flat surface is equal to *twice* the mixed partial derivative of the surface height *z*:

$$a_{45} = 2 \cdot \frac{\partial^2 z}{\partial x \partial y} = 2 \cdot g \cdot x \qquad [9]$$

Differentiating this expression with respect to *x* provides the rate of change in surface astigmatism at axis 45 degrees *lateral* to the umbilic of the elephant trunk surface:

$$\frac{\partial a_{45}}{\partial x} = 2 \cdot g \qquad [10]$$

where *g* is the rate of change in surface curvature *along* the umbilic—that is, the third partial derivative of the surface with respect to *y* ($\partial^3 z/\partial y^3$). Further, the rate of change in *cylinder power* δ*Cyl*, in diopters, is related to the rate of change in surface astigmatism at axis 45 degrees by a factor of 1000 (*n* – 1), so that:

$$\delta Cyl = 2 \cdot g \cdot 1000 \cdot (n-1) \qquad [11]$$

Finally, substituting the power law relationship for *g* yields Equation 8:

$$\delta Cyl = 2 \cdot \frac{\delta Add}{1000 \cdot (n-1)} \cdot 1000 \cdot (n-1) = 2 \cdot \delta Add$$

Ultimately, Minkwitz's theorem demonstrates that it is not possible to produce a change in "spherical" addition power *along* the progressive corridor without introducing surface astigmatism *away* from the corridor. Further, Minkwitz's

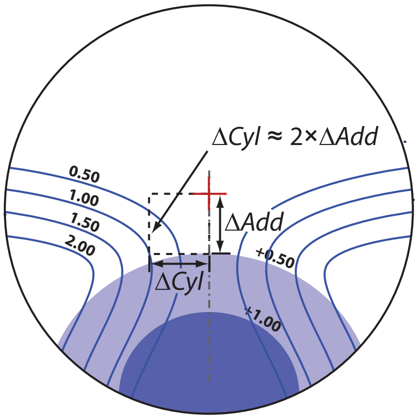


Figure 6. Minkwitz's theorem implies that the change in cylinder power (Δ*Cyl*) at a small distance away from the corridor is roughly equal to twice the change in addition power (Δ*Add*) at an equal distance along the corridor.

theorem implies that the change in cylinder power at a small distance away from the corridor is roughly equal to twice the change in addition power at an equal distance along the corridor, as illustrated in Figure 6. Minkwitz's theorem also provides some other useful insights into the nature of progressive optics. The average rate of change in addition power is *directly proportional* to the addition and *inversely proportional* to the corridor length of the lens design. Therefore, with the application of Minkwitz's theorem, two important guidelines regarding the optics in the central regions of a progressive lens surface can be deduced:

1. The rate of change in cylinder power away from the umbilic increases as the addition power of the lens increases. This means that the unwanted cylinder power in the periphery of the lens design is roughly proportional to the addition of the lens (Figure 7).
2. The rate of change in cylinder power away from the umbilic increases as the length of the corridor decreases. This means that lens designs with shorter corridor lengths produce more unwanted cylinder power in the periphery or narrower viewing zones (Figure 8). Because of the more rapid increase in cylinder power, the width of the progressive corridor is also reduced.

Characterizing Progressive Optics

There are a variety of useful metrics to evaluate when assessing the performance of a progressive lens design. Recall that, while the central regions of a progressive lens surface are *nearly spherical*, astigmatism exists at most points across the lens surface. Each point across the lens surface can therefore be represented locally as a combination of unwanted astigmatism (or cylinder power) and addition power—or, more specifically, *mean* (average) addition power in the presence of astigmatism. Unwanted astigmatism and mean addition power are the most common optical quantities to assess when characterizing or evaluating the optics of progressive lenses.[10,11]

Contour plots, which are maps indicating how the levels of a given quantity vary across a surface, are particularly convenient for representing the distribution of astigmatism, add power, and other optical quantities across a progressive lens (Figure 9). Astigmatism contour plots indicate regions of potential blur and distortion, and are therefore useful for predicting the size of the distance, intermediate, and near zones of the lens design as well as the utility of the periphery. In particular, the usable width of the central viewing zones of a progressive lens is often delimited by the 1.00-diopter astigmatism boundaries.[12]

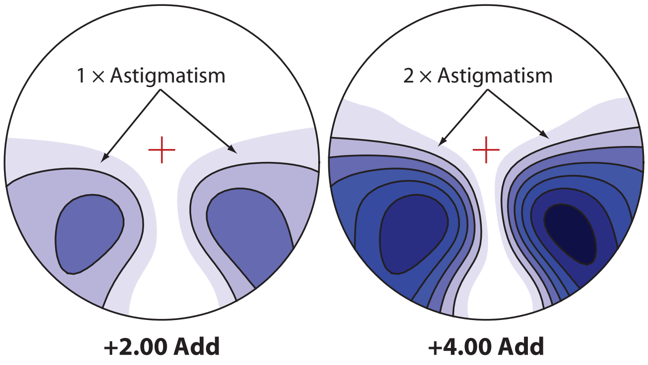


Figure 7. As a consequence of Minkwitz's theorem, the unwanted surface astigmatism and cylinder power in the periphery of a progressive lens is roughly proportional to the add power of the lens.

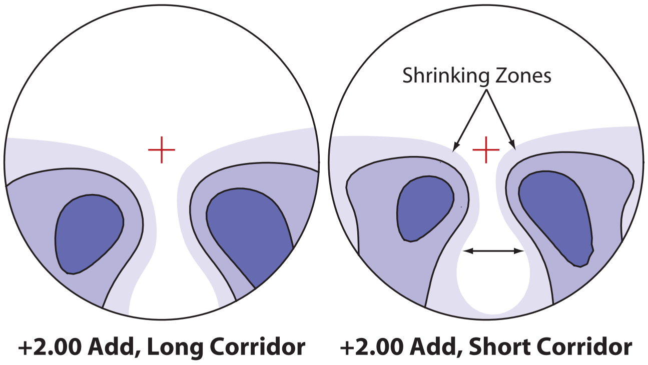
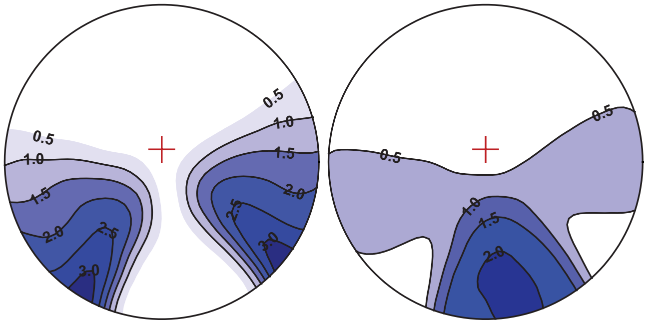


Figure 8. As a second consequence of Minkwitz's theorem, shorter progressive corridor lengths produce greater levels of unwanted cylinder power, smaller viewing zone sizes, or a combination of both.



Astigmatism Plot **Mean Add Power Plot**

Figure 9. Contour plots show the distribution of an optical quantity—such as unwanted astigmatism (cylinder power) or mean add power—across the lens by indicating the magnitudes of the quantity at fixed intervals (for example, 0.50 diopters).

Typically, each progressive lens has a unique astigmatism contour plot, so these plots also serve as a kind of "fingerprint" of the lens design. Mean add power contour plots, on the other hand, indicate the size and location of the near zone as well as regions of excess plus power that may contribute to blur during far vision.

Plots of surface power provide a convenient way to evaluate the optics of a lens design, but they are only *indicative* of performance. Furthermore, plots of *surface* quantities are usually less meaningful than plots of *ray-traced* optical performance, which typically rely on modeling the lens in the "position of wear" in order to determine how the *wearer* actually perceives the optics of the lens. The *position of wear* represents the intended position of the glazed and fitted spectacle lens relative to the visual system of the actual wearer, including the vertex distance and any lens tilt.

Although plots of surface astigmatism and mean add power are the most common measures of optical performance, they fail to represent the *combined* interaction of these effects upon vision. Both unwanted cylinder power *and* excess—or insufficient—addition power contribute to blur. *RMS* (root-mean-square) *power* combines both the astigmatic and mean power errors into a single measure of power. RMS power is a more clinically meaningful measure of optical performance, and a useful predictor of blur and visual acuity.[13] It is also possible to characterize the optics of a progressive lens using *wavefront* analysis. Wavefront analysis evaluates "high-order" aberrations of the lens in addition to the "low-order" aberrations represented by astigmatism and excess addition power (or defocus). The significance and application of wavefront analysis in progressive lens design will be described in detail in the second part of this series.

Modern Progressive Lenses

Progressive lens design and manufacturing techniques have improved considerably since the early lenses of the 1960s. Significant technological advancements over the past four decades have provided progressive lens manufacturers with sophisticated tools to design and fabricate progressive lenses. The introduction of numerically-controlled cutting, either to grind glass lens surfaces directly or to shape refractory materials suitable for "slumping" glass at high temperature, has eliminated most manufacturing limitations, while the introduction of high-speed computing has made possible lens designs of virtually unlimited complexity. Today, progressive lens designers are constrained only by the inherent mathematical limitations of these surfaces.

Although early progressive lenses were quite crude in design, and enjoyed only limited success, modern progressive lenses generally perform quite well for most spectacle wearers. In fact, numerous studies have demonstrated that progressive lenses are now preferred over conventional bifocal lenses by the vast majority of subjects.[14,15] It has even been estimated that progressive lenses are preferred to conventional bifocal lenses by roughly four to one.[16]

Distribution of Power and Astigmatism

No longer faced with the limitations imposed by early lens design and manufacturing techniques, lens designers have been free to pursue more generalized surfaces of greater complexity. Improvements to early progressive lens designs focused on reducing unwanted astigmatism in the periphery to its mathematical limits while better managing the overall distribution of addition power and astigmatism across the lens surface. This could be accomplished both by varying the horizontal curvatures of the surface appropriately and by carefully managing the progression of addition power along the umbilic.

The horizontal cross-sections of the basic progressive lens model presented earlier are essentially circular, resulting in an extremely rapid increase in unwanted astigmatism in the periphery of the lens surface, particularly when a non-linear power law is utilized. This was especially problematic for many early progressive lenses, which often concentrated unwanted astigmatism into relatively small regions of the lens periphery. The use of non-circular, aspheric cross-sections, including *conic* sections that varied in eccentricity down the corridor, reduced the rapid increase in surface astigmatism in the periphery while allowing some of the surface astigmatism to be distributed into the distance periphery without overly compromising the utility of the central distance viewing zone.[17,18]

The eventual use of "spreading" or "smoothing" functions further reduced levels of surface astigmatism to its mathematical limits, while also providing considerable freedom in defining the viewing zone configuration of the lens design. One such approach applied *Dirichlet's principle*, or the principle of minimum potential energy, to the problem of distributing power and astigmatism in the smoothest possible way between the distance and near zones by minimizing a Dirichlet integral.[19] For "modern" progressive lenses, the peak level of unwanted cylinder power seldom exceeds the magnitude of the addition power of the lens by more than 20 percent.

In addition to developing novel lens surfaces with minimal unwanted astigmatism, lens designers also began investigating the optimal distribution of surface optics across the lens. The spatial distribution and rates of change—or *gradients*—of power and astigmatism across the surface are fundamental aspects of the lens design that define the gross optical performance of the lens. Often, progressive lens designs are broadly categorized as either "hard" type designs or "soft" type designs based upon the distribution of power and astigmatism (Figure 10):

- *Hard* type lens designs concentrate the progressive optics into smaller regions of the lens surface, thereby expanding the areas of clear vision at the expense of elevating the gradients and overall magnitude of unwanted cylinder power in the periphery. Because of this, harder progressive lenses generally offer wider distance and near viewing zones, but higher levels of blur and distortion in the periphery. Hard designs will generally work better for sustained viewing tasks requiring good visual acuity, and tend to offer the kind of utility that current bifocal wearers enjoy.
- Soft type lens designs spread the progressive optics across larger regions of the lens surface, thereby reducing the gradients and overall magnitude of unwanted cylinder power at the expense of narrowing the areas of clear vision. Because of this, softer progressive lenses generally offer less blur and distortion in the periphery, but narrower viewing zones. Soft designs will generally work better for dynamic viewing tasks, and tend to improve visual comfort and adaptation for emerging presbyopes.

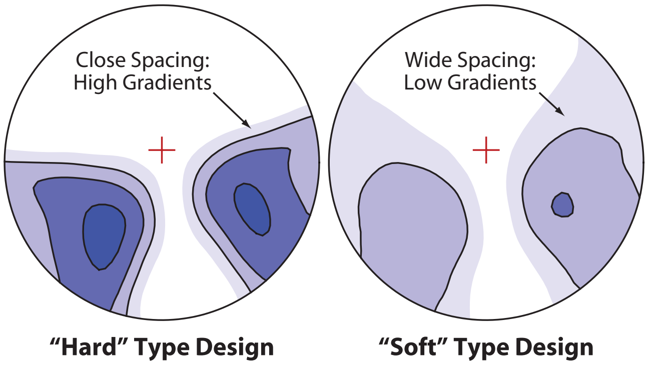


Figure 10. Unwanted astigmatism is spatially distributed over much of the lens surface in "soft" type designs, which therefore exhibit relatively low gradients of astigmatism, whereas unwanted astigmatism is confined to smaller regions of the lens surface in "hard" type designs, which therefore exhibit relatively high gradients of astigmatism.

Essentially, the gradients of surface power and astigmatism across the lens design must increase as the area of the lens surface used to "blend" the distance and near zones is decreased. Since the overall utility of the lens design relies on a careful balance between clarity of vision and visual comfort, modern progressive lenses are seldom strictly "hard" or "soft" in design, but instead represent a well-considered compromise between these two approaches. It is equally important that the relative balance between the distance zone size and the near zone size reflect the typical wearer's use of the lens. Lens designers often seek to find the best overall balance between the utility of the three central viewing zones and the periphery of the lens.[20]

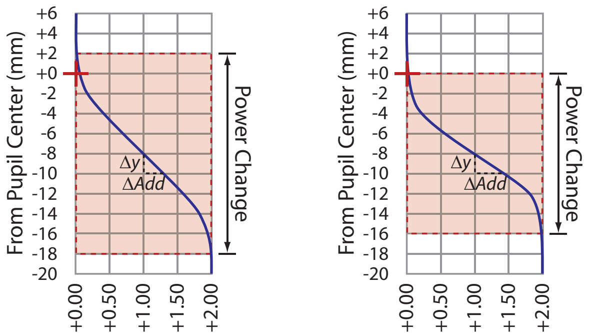
The distribution of power and astigmatism across the lens surface may be tuned differently for different addition powers, as well as for different base curve and addition power combinations. For instance, the progressive lens may employ a "softer" lens design with a longer corridor length for low additions and a "harder" lens design with a shorter corridor length for high additions, or vice versa, depending upon the design strategy. In some cases, the progressive lens design may vary the size of the central viewing zones by base curve in order to provide more consistent fields of view by accounting for the effects of spectacle magnification. Lens designs that vary as a function of addition power are referred to as *multi-design* lenses, whereas lens designs that vary as a function of both base curve and addition power are sometimes referred to as *design by prescription* lenses.

The Power Profile

Since the surface of a progressive lens is very nearly spherical in the vicinity of the umbilic, the optical performance of the central viewing zones of a progressive lens is largely dictated by the progression of addition power along the progressive corridor (that is, the power law), in accordance with Minkwitz's theorem. *Soft* type lens designs typically utilize a longer progressive corridor length with a relatively slow progression of addition power, whereas *hard* type lens designs typically utilize a shorter corridor length with a relatively rapid progression of addition power. A graph of addition power as a function of the vertical position within the progressive corridor is known as the *power profile* of the lens design, as shown in Figure 11.

The ergonomic utility of the lens design for many viewing tasks depends upon carefully locating the distance and near zones in order to minimize unnecessary head and eye movements while ensuring clear, comfortable vision during both sustained and dynamic viewing tasks. Ideally, the design of the power profile should reflect the wearer's typical use of the near and intermediate zones for reading and mid-range viewing tasks while minimizing unwanted blur from excess plus power within the central distance zone. The length of the progressive corridor should represent a sensible balance between the various tradeoffs involved:

- *Shorter* corridor lengths afford the wearer with a more readily accessible near zone and sufficient reading utility across a wider range of frame sizes and fitting heights. Since every one millimeter of corridor length at the spectacle plane necessitates roughly two degrees of additional ocular rotation to reach the near zone, a shorter corridor length requires fewer—potentially awkward—postural adjustments.
- *Longer* corridor lengths afford the wearer with greater mid-range utility and either wider viewing zones or lower levels of unwanted astigmatism in the periphery. Since the rate of change in cylinder power is proportional to corridor length, a longer corridor length may improve dynamic vision and overall wearer comfort.



"Soft" Type Power Profile **"Hard" Type Power Profile**

Figure 11. The optics of the central viewing zones and the overall ergonomic utility of the lens design for many viewing tasks are largely dictated by the length of the corridor and the shape of the power profile. Note that, at any point along the corridor, the power law (δ*Add*) is given by Δ*Add* ÷ Δ*y*.

The lens design should also afford sufficient near utility across a wide range of frame styles. Progressive lenses first became popular in the 1970s and 1980s—decades that represented the peak of large spectacle frame styles. The capacious frame styles in vogue during these early decades afforded the typical progressive lens wearer with more than sufficient vertical clearance for the intermediate and near zones of traditional progressive lens designs. By the 1990s, however, the fashion trend in frame styles was decidedly "minimalist," with frame dimensions shrinking dramatically.

Eventually, progressive lenses designed specifically for smaller frame styles were introduced.[21] This new class of progressive lenses utilizes significantly shorter corridor lengths that afford lower minimum fitting heights. Of course, reducing the corridor length of the lens design necessitates various optical compromises, in accordance with Minkwitz's theorem. Since shorter corridor lengths result in smaller viewing zone sizes or higher levels of unwanted cylinder power in the periphery, lens designers must carefully manage the optics of these designs in order to ensure sufficient visual utility.

Binocularity

Early progressive lenses designs were completely *symmetrical* with respect to the umbilic. The desired near zone inset for near vision was achieved mechanically by simply having each lens rotated by nine degrees or more. In effect, the same lens blank could be used for either eye prior to surfacing. This process would rotate the more deleterious optics of the surface, however, into the upper nasal (medial) quadrant of each glazed lens. Since excess addition power, unwanted cylinder power, and prism then differed between the nasal and temporal regions of each lens, binocular vision was significantly disrupted as the wearer gazed laterally across the lenses. Further, although vision through the temporal field of each lens was left unobstructed, the

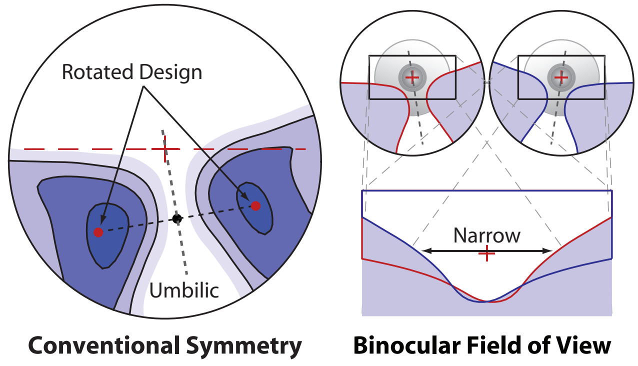


Figure 12. "Symmetrical" progressive lens designs are rotated to achieve the desired inset for near vision, which disrupts binocular fusion and limits the binocular field of clear vision.

binocular field of view was restricted by excess blur in the nasal field of the opposite lens (Figure 12).

Eventually, lens designers began altering the design of the lens on either side of the umbilic in order to achieve the desired near zone inset optically, instead of mechanically. *Asymmetric* lens designs were an early application of this concept; these designs essentially constrained the nasal surface astigmatism below a fixed horizontal axis as the umbilic was effectively rotated nasally.[22] By designing the path of the umbilic with an optical inset, better alignment could be obtained between the right and left viewing zones during binocular vision, maximizing the binocular field of view. Nevertheless, while asymmetric lens designs increase the binocular field of view through the lenses, the nasal surface astigmatism of these lens designs is often considerably higher than the temporal astigmatism, since the astigmatism becomes more "concentrated" as the near zone is effectively rotated into the nasal region.

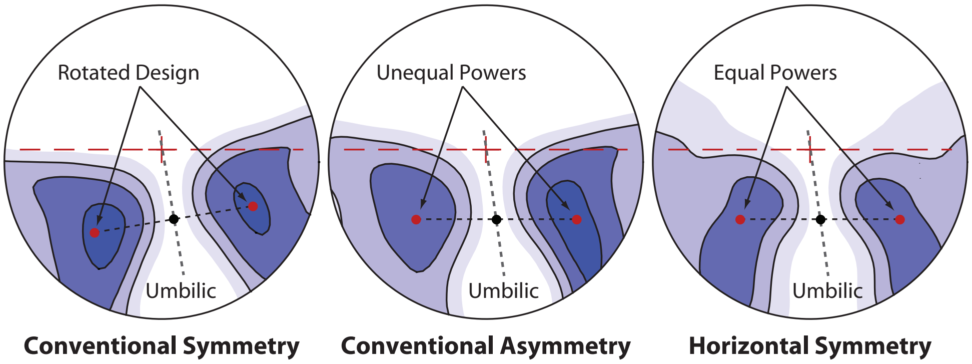
The next innovation in improving binocular vision performance was obtained by more carefully balancing the optics to either side of the umbilic. *Horizontal symmetry* ensures that the power, prism, and magnification remain relatively equal for corresponding points across the right and left lenses during binocular vision, so that magnification disparities and prismatic imbalance are minimized, ensuring better binocular fusion (Figure 13).[23] This also reduces the stereoscopic distortion of space that can occur when significant differences in magnification exist between the two lenses. Additionally, many modern lens designs often vary the inset of the near zone for each base curve and addition power combination to account for the effects of both shorter working distances with higher additions and any prism induced by the distance prescription during convergence.

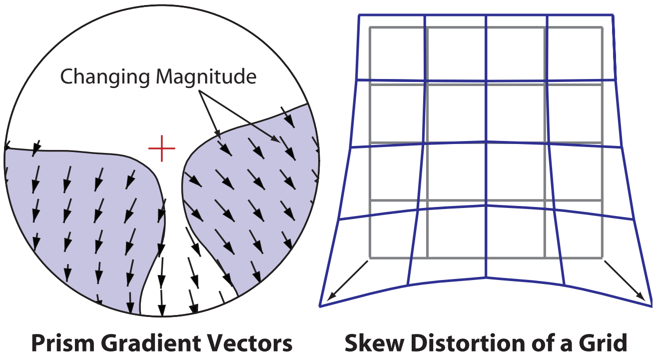
Design of Periphery

The inherent surface astigmatism and rapid changes in power and prism in the peripheral "blending" regions of progressive lens designs produce several optical phenomena that may be visually disturbing to the wearer initially, particularly under dynamic viewing conditions. Fortunately, much progress has been made over the past few decades in minimizing these "optical side-effects" by better managing the optical design of the lens periphery. With more sophisticated lens design tools and a better understanding through vision research of the most visually significant imaging defects, progressive lens designers have been able to minimize rapid undulations in power and prism and to achieve better overall *orthoscopy*, or lack of geometric distortion, in the periphery of the lens.

Recall that the cylinder power in the periphery of a progressive lens is generally oriented at a highly oblique axis. This unwanted cylinder power produces differential spectacle magnification at a similar orientation. This optical imaging defect is known as *skew distortion*, and causes objects—such as straight edges—to appear tilted, sheared, or even curved (Figure 14). Minimizing skew distortion and improving orthoscopy can be achieved by orienting the surface astigmatism more vertically or by reducing the

Figure 13. The methods utilized to obtain a near zone inset in progressive lenses have improved significantly over the years, with asymmetric lens designs increasing the width of the binocular fields of view compared to symmetrical designs and horizontally symmetric lens designs reducing power, prism, and magnification differences that could impair binocular fusion.





Prism Gradient Vectors **Skew Distortion of a Grid**

Figure 14. The presence of cylinder power at an oblique axis in the periphery of a progressive lens design, combined with rapid variations in power and prism, can result in skew distortion and visual discomfort if not carefully managed.

overall magnitude of the astigmatism, since either will reduce the astigmatism component at axis 45 degrees.

Additionally, the visual field flow is artificially modified by optical prism gradients across the progressive lens. Variations in prism and magnification cause an apparent acceleration of stationary objects that differs from the physical movement detected by the sensorimotor system of the wearer. This optical imaging defect is known as *image swim*. The neurophysical system for detecting physical movement includes the *vestibular apparatus*, which is linked to the visual system and plays a major role in maintaining balance and stabilizing vision while in motion. When significant image swim is present, causing objects to appear to “rock” or sway unnaturally, the wearer’s visual perception of movement may conflict with the vestibulo-ocular reflex, inducing a sensation of vertigo or motion sickness.

Numerical Optimization Methods

Another advancement in progressive lens design was the introduction of numerical optimization methods to fine-tune the optical performance of the lens. In a typical application of this technology, an initial starting surface is first defined and then modeled mathematically using a finite element method. The computational area of the lens surface is "discretized" by breaking regions of the surface up into square elements across a reference grid or *mesh*. The intersection points across the mesh that define these square elements are referred to as *nodes*. Each node has an array of mathematical quantities associated with it that characterize the surface at that point, including the local curvatures. These nodes are mathematically joined using basis functions known as *bivariate splines*, which ensure a continuously smooth surface.

A target distribution of optical quantities, representing the ideal distribution of characteristics such as mean power and astigmatism, is specified for each node location across the lens surface. Generally, a smooth surface cannot achieve this target distribution, at least for every point. Finite element method seeks to minimize the difference between the desired optical performance at any point on the surface and the actual optical performance possible with a continuously smooth surface. This is accomplished by minimizing *merit functions* at each node, which are equations used to find least-squares solutions of the form:

$$M = \sum_{i=1}^{n} w_i \cdot \left(A_i - T_i\right)^2$$

…*Merit function* [12]

where *M* is the value of the merit function to minimize at a given node location, A_i is the actual value of the measurement, T_i is the target value of that measurement, and w_i is the weighting factor assigned to the measurement quantity at a given node. The merit functions at each node location are integrated across the entire lens surface.

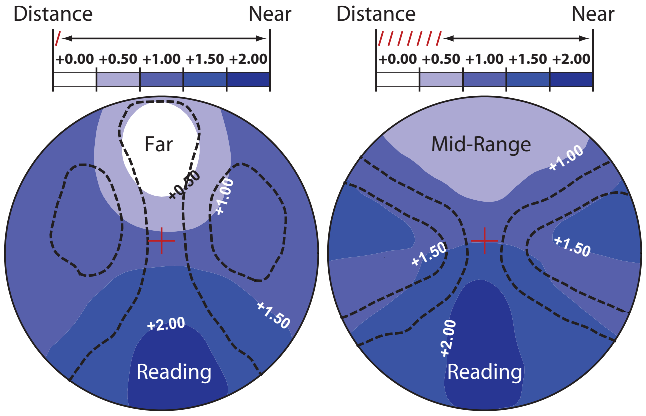
Common quantities to minimize include power errors, unwanted astigmatism, gradients of power, and so on. The weightings (w_i) for these quantities can vary spatially as a function of node location, allowing different regions of the lens surface to emphasize different performance attributes. The central viewing zones, for instance, are generally more heavily weighted, so that the analysis achieves more precise optical control in these regions. These measurement quantities may be calculated directly from surface characteristics or, alternatively, derived from ray tracing a lens-eye model for the position of wear.

Occupational Progressive Lens Designs

Although this article has focused mainly on “general-purpose” progressive lenses, there is also a class of progressive lenses designed with an emphasis on mid-range and reading vision. These “occupational” progressive lens designs are particularly suited to computer use and other demanding viewing tasks characteristic of an office environment. The stress upon the visual system resulting from intensive, prolonged reading and computer use may contribute to a variety of symptoms that are often associated with *computer vision syndrome*, or the complex of eye and vision problems related to near work and computer use. Without proper optical correction, this stress may elicit symptoms associated with accommodative dysfunction (for example, blurred vision or slowness of refocusing), asthenopia (for example, eyestrain or headache), and even musculoskeletal strain (for example, neck and back pain).[24] In fact, one investigation of clinical studies pertaining to the prevalence of computer vision symptoms concluded that 50 percent or more of computer users complain of some form of eye problems associated with computer use.[25]

The viewing zone configuration and range of addition power offered by occupational progressive lenses reflect the more sedentary visual demands of typical office and computer work, providing very little—if any—far vision utility. These progressive lenses are generally characterized by exceptionally wide intermediate and near zones, often combined with a marked reduction in unwanted astigmatism in the periphery of the lens design. Of course, the wider intermediate and near viewing zones and reduced unwanted astigmatism are achieved at the expense of the distance zone. Further, these lenses typically offer a smooth power law, or rate of change in addition power, by utilizing a relatively long progressive corridor and by reducing the total change in addition by starting the addition at an intermediate power intended for mid-range working distances. Occupational progressive lenses are sometimes categorized as either “computer progressive lenses” or “enhanced single vision lenses,” which are distinctions that reflect the extent of the range of clear vision typical of each lens design as well as the overall design strategy (Figure 15).

Computer progressive lenses are similar to traditional general-purpose progressive lenses, and may even offer some degree of far vision utility.[26] The distance zone of these lenses is typically smaller and higher compared with general-purpose progressive lenses, and some designs provide a low addition within the distance zone (for example, +0.50 diopters), which still allows for mobility indoors. These lenses are generally available in a full range of additions and base curves, and are also fitted like traditional progressive lenses. *Enhanced single vision lenses*, on the other hand, typically provide only mid-range and reading utility, but frequently offer wider intermediate and near zones that are more readily accessible.[27] These lenses are generally available in only one or two possible power changes—each associated with a range of prescribed additions—and are fitted like either progressive lenses or single-vision lenses, depending upon the recommendations of the manufacturer.



“Computer Progressive” **“Enhanced Single Vision”**

Figure 15. “Computer progressive lenses” typically provide a full range of addition power—less perhaps a low “indoor” correction—for some degree of far vision utility, whereas “enhanced single vision lenses” typically provide a limited range of add power intended for mid-range and reading utility, only.

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Progress in the spectacle correction of presbyopia.
Part 2: Modern progressive lens technologies

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The first installment of this two-part series reviewed the fundamental optical principles and early development work associated with progressive lenses. Recent progress made in advancing the state of the art in progressive lenses will now be presented, with particular emphasis on "free-form" progressive lenses and the application of "wavefront" technology in progressive lens design. Because several fundamental concepts were developed in the first paper that will serve as the basis for discussions presented in this paper, including the basic optics and mathematics of progressive lens surfaces, the reader is strongly encouraged to review the companion paper.

Key words: lens design, presbyopia, progressive lenses, spectacle correction

Limitations of Traditional, Semi-Finished Lens Design

Modern progressive lens designs work quite well for the majority of wearers, with acceptance rates of 90% or more. Ongoing vision research continues to make incremental advancements in progressive lens design by providing lens designers with greater insights into the optical qualities most critical to presbyopes. In fact, lens designers may very well be approaching a "limiting" class of progressive lens designs that represent the best overall balance of optical characteristics necessary to maximize visual utility for the *average* progressive lens wearer. Nevertheless, the visual requirements of spectacle lens wearers vary from person to person, and it has long been understood that traditional, "one-size-fits-all" progressive lenses will not be the ideal solution for *every* progressive lens wearer.[1]

By considering the unique visual requirements of the *individual* progressive lens wearer, on the other hand, the optics of the lens design can be more suitably tailored to each wearer, maximizing wearer satisfaction. Nevertheless, the economics of offering mass-produced, *semi-finished* (that is, factory-fabricated) progressive lens blanks in multiple design variations are prohibitive. Each lens design typically requires sixty or more different base curve and addition power permutations per eye in up to twelve different lens materials, which necessitates massive product development and inventory costs. Therefore, changes to the basic lens design have been limited to subtle variations in the optical design of each base curve and addition power combination that must work sufficiently well for the entire prescription range associated with that particular lens blank. Moreover, since semi-finished lenses are typically limited to a handful of base curve options because of these inventory constraints, optical performance is ultimately compromised for many prescriptions.

"Free-Form" Progressive Lenses

Fortunately, the advent of "free-form" technology has freed many lens designers from the constraints of traditional mass lens production by enabling a local prescription optical laboratory to deliver progressive lenses designed and produced in "real time" for a specific wearer. *Free-form surfacing* is simply a manufacturing platform that allows the production of complicated lens designs in a small-scale production environment "on demand." Until now, progressive lenses had been relegated to a highly involved, mass production environment. Free-form surfacing has made possible the production of complex lens designs on a per-job basis at the laboratory level, on the other hand, by providing laboratories with the means to surface progressive and other complicated lens designs directly onto a lens blank.

The inherent visual benefit of progressive lenses produced using free-form surfacing is minimal compared with similar lenses produced using traditional lens casting and surfacing. Although the free-form surfacing process may arguably offer more precise replication of progressive lens designs, this benefit relies on meticulous process engineering in order to ensure lens surfaces of consistently good quality and accuracy. Traditional lens casting, on the other hand, is a highly repeatable process that delivers relatively consistent quality, albeit with some loss of fidelity in reproducing certain lens design features due to factors such as shrinkage while the liquid monomer polymerizes. Furthermore, although the precision of free-form surfacing is not limited by the availability of hard lap tools, often stocked in only tenth- or eighth-diopter increments, these lenses are still held to typical optical tolerances and subject to manufacturing variances, particularly in the absence of adequate process engineering.

When used in conjunction with sufficiently advanced lens design software, however, a free-form delivery system can produce a completely arbitrary progressive lens design that has been fully parameterized using input specific to the individual wearer. Consequently, if the visual and optical requirements of a particular wearer are known prior to the optical design stage, it becomes possible to customize the design of the progressive lens accordingly. Alternatively, since free-from surfacing is not subject to the inventory constraints of semi-finished lenses, a suitable progressive lens may be selected from a range of possible lens designs, thus allowing for a greater degree of freedom in matching the lens design to the specific wearer. Therefore, as a "technology enabler," free-form surfacing can serve as a critical vehicle to deliver considerable visual benefits to the wearer. When the potential of individualized progressive lens production via free-form surfacing is fully realized, optical performance and wearer satisfaction are maximized.

It is also possible to utilize free-form surfacing to deliver traditional-type progressive lenses on demand, often by mathematically combining a "fixed" progressive lens design from a predefined surface description file with the prescription sphere and cylinder curves normally applied to the back of the lens blank.[2] Since the progressive lens design may be surfaced directly onto the back of the lens blank along with the prescription curves, only a small range of "pucks," or semi-finished lens blanks with spherical front surfaces corresponding to the desired base curves, is necessary for lens production, thus obviating the need for a large inventory of semi-finished progressive lens blanks. Although there may be a minor reduction in certain unwanted magnification effects, free-form progressive lenses of this type essentially replicate the performance of traditional lenses made from mass-produced, semi-finished progressive lens blanks. Consequently, one should distinguish between so-called "smart" free-form progressive lenses that are truly designed for the wearer in real time and free-form progressive lenses that are produced directly from surface description files with little optical modification for the wearer, if any.[3]

Free-Form Lens Surfacing

A "traditional" lens surfacing process cannot produce the complex surfaces utilized for complicated lens designs like progressive lenses due to limitations in both the range of possible geometries and the "quality" of surfaces produced by conventional generators. Conventional generators were designed with an emphasis on efficient stock removal from simple spherical and toroidal surfaces of revolution, which can be smoothed and polished using rigid (that is, "hard") lap tools of similar curvature in combination with various abrasives. However, unlike these basic surfaces of revolution, complex progressive surfaces must be smoothed and polished with flexible (that is, "soft") lap tools, since the curvature does not remain constant across the surface.

The accuracy and finish of a machined surface is generally evaluated for several different qualities, including *surface roughness* prior to polishing and errors from the desired shape, or *form*, including *waviness* (Figure 1). Conventional, two-axis generators can produce only simple surfaces of revolution. Newer, three-axis generators were not designed to produce complex lens surfaces to the level of precision and smoothness required for soft lap polishing. The surface roughness off both two-axis and three-axis generators is still relatively high, and often comparable in magnitude to the errors in form necessary to create visible optical effects, such as "waves." These generators rely on hard lap tools affixed with abrasive pads to correct errors in form and

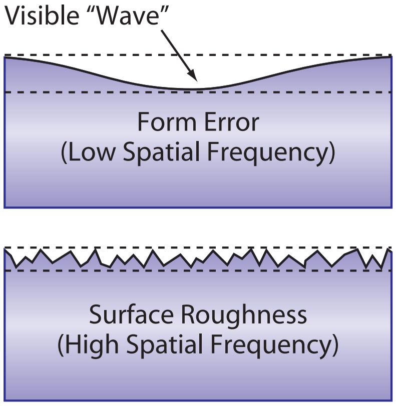


Figure 1. The finish quality of a machined surface is often evaluated in terms of surface roughness (or "high" spatial frequency errors) prior to polishing, whereas the accuracy of the surface is often evaluated in terms of form errors (or "low" spatial frequency errors) or waviness.

curvature while bringing the surface to a level of smoothness suitable for polishing.

A "free-form" lens surfacing process, on the other hand, can produce even highly complex surfaces like progressive lens designs in a matter of minutes. *Free-form generators* are highly sophisticated machines capable of producing very precise surfaces of high complexity using a computer-controlled, single-point cutting process (Figure 2). *Free-form polishers* utilize a flexible, computer-controlled "soft lap" tool capable of polishing the complex lens surfaces produced by free-form generators. Common free-form generators utilize single-point diamond turning, with a combination of diamond tools, to produce accurate surfaces of sufficient smoothness that require only a short polishing cycle using a soft lap tool, since excess polishing can distort the surface of the lens.

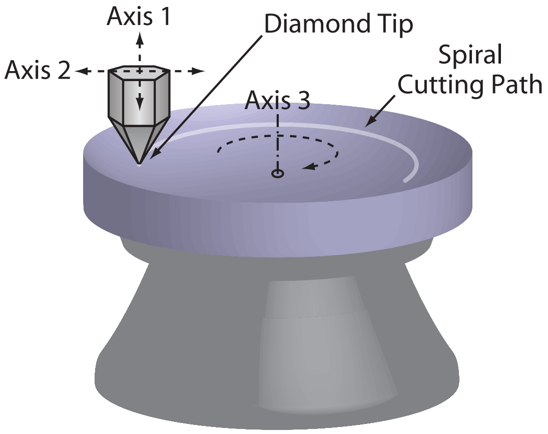


Figure 2. Free-form generators use precise, computer-controlled cutting techniques, such as single-point diamond turning, which are capable of producing complex lens surfaces with considerable accuracy and smoothness.

The 1970s saw the first commercial applications of computer-numerically-controlled (CNC) machines for shaping parts. Over the past ten years, in particular, improvements in machine stiffness, encoder resolution, and controller bandwidth have yielded free-form generators that produce exceptionally smooth, precise surfaces that now sufficiently replicate most progressive lens designs. Moreover, although free-form surfacing equipment was extremely expensive, few in number, and largely restricted to precision optics applications in the past, more affordable free-form production cells are now available, making this technology a viable manufacturing platform for many prescription optical laboratories.

A typical free-form surfacing process begins by first mathematically modeling a lens surface. Most commonly, this surface represents the combination of a progressive lens design with the required prescription curves, which will be surfaced onto a spherical "puck." In a sufficiently advanced process, this lens surface may also be optically modified using various parameters specific to the wearer.[4] Alternatively, the surface may represent optically-optimized (or "atoric") prescription curves only, which will be surfaced onto a semi-finished progressive lens blank with the progressive lens design prefabricated on the front surface.[5]

The final surface is then rendered as a digital cutting file, or "points" file, which is transmitted to the computer controller of the free-form generator. The back surface of a semi-finished lens blank with a prefabricated front surface is then subjected to a three-stage cutting process by the generator, which utilizes a multi-blade tool for rough cutting, a polycrystalline diamond tool for smooth cutting, and a natural diamond tool for a high quality finishing pass. After generating, the lens blank is transferred to a free-form polisher, where it undergoes a computerized polishing process that utilizes a dynamically-controlled, soft lap tool made from a compliant foam or similar material.

Prescription Customization

As Figure 3 illustrates, each prescription power requires a unique "best form" base curve or aspheric lens design in order to eliminate optical aberrations such as *oblique astigmatism*.[6] The first commercial "best form" lenses utilized a separate base curve for every power in order to maximize optical performance for every power in the prescription range.[7] Modern semi-finished lenses, however, generally have relatively broad prescription ranges grouped

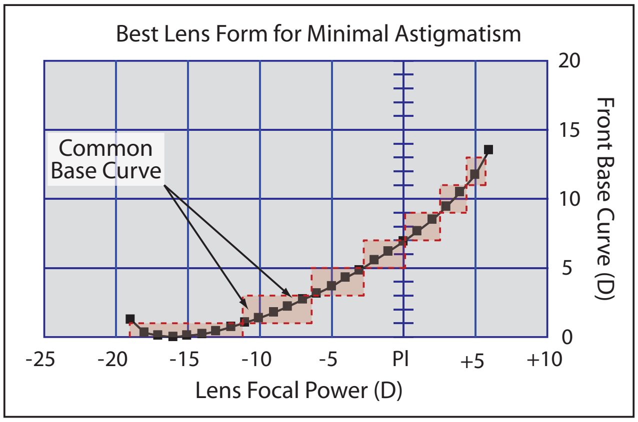


Figure 3. Although modern semi-finished progressive lenses have broad prescription ranges grouped upon a limited number of base curves, "best form" optical principles dictate that each lens power ideally requires a unique base curve or aspheric lens design in order to eliminate optical aberrations such as oblique astigmatism.

upon a limited number of common base curves, which compromises optical performance for many prescriptions. Additionally, while the use of a unique lens design may satisfy the optical requirements for spherical prescriptions, a conventional lens surface cannot simultaneously eliminate the aberrations produced by both the sphere and cylinder meridians of lenses made with sphero-cylindrical prescriptions.

While each individual base curve performs optimally for a single, spherical lens power, as the prescription deviates further and further from this "optimal" power, the zones of clear vision become restricted as the residual lens aberrations worsen. Factors such as lens tilt and prism introduce additional lens aberrations and blur. Residual lens aberrations are of even greater importance with progressive lenses, since any oblique astigmatism will interact with the unwanted astigmatism of the progressive surface. The resulting cross-cylinder effects can cause the clear zones of vision to shift, shrink, or rotate as regions of the lens designed to be clear become blurred, while certain regions of blur actually become clearer to the wearer. These effects reduce the utility of the progressive lens design under both monocular and binocular viewing conditions.

In semi-finished optical design, the application of numerical optimization methods or asphericity can maximize the optical performance of the lens design for a single prescription, which generally corresponds to the median sphere power of the prescription range associated with each base curve. On the other hand, if the wearer's specific prescription requirements are known before the lens is actually designed, these prescription exact values can be utilized, instead, during the optimization process. By precisely matching the design of the lens to the intended prescription, excess lens aberrations are eliminated, and the ideal performance of the progressive lens design is preserved. Fortunately, the individualized approach to lens manufacturing afforded by free-form technology makes this possible.

Prescription customization represents the application of numerical optimization methods or asphericity to a free-form lens that is designed in "real time" using parameters specific to the individual wearer. Advanced prescription optimization techniques generally seek to find the "optimum" surface that minimizes the differences between the actual performance of the lens design and the ideal, target performance. This is done by manipulating the initial surface until a merit function is minimized that represents a variety of appropriately weighted optical and geometric properties, including the distributions of power and unwanted astigmatism. The net result of this optimization process is a complex "aspherization" of the initial progressive lens surface that achieves the ideal, "best form" optical performance requirements across the viewing zones of the lens, regardless of the base curve of the lens blank or the specific prescription (Figure 4).

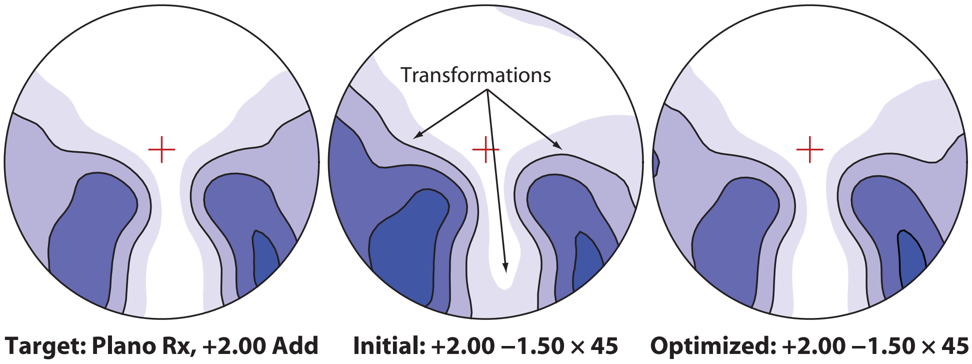


Figure 4. A sophisticated prescription optimization process, used in conjunction with free-form lens surfacing, can achieve the ideal performance of the lens design for virtually any prescription, as demonstrated by these plots of ray-traced optical astigmatism. Note the distortion of the viewing zones that occurs due to the prescription in the absence of optimization.

Prescription Compensation

Accurate prescription optimization relies on ray tracing a lens-eye model for an assumed *position of wear*, which represents the intended position of the fitted spectacle lens with respect to the visual system of the actual wearer. Like conventional bifocal lenses, traditional progressive lenses are designed to provide the correct (that is, the prescribed) "vertex" powers at the distance and near power verification points when measured using a standard focimeter. In this case, the lens is held with the back surface normal to the axis of the instrument—often coincident with the optical axis of the lens. This measurement geometry nicely replicates the position of the trial lenses used during ocular refraction, as well. However, since the spectacle lens is generally positioned in a very different fitting geometry relative to the optics of the eye, the effects of vertex distance, lens tilt, and even viewing distance influence the optical powers of the lens as experienced by the actual wearer (Figure 5).

The effects of vertex distance on lens power are generally well understood. Tilting a lens produces a form of *oblique astigmatism* that introduces unwanted cylinder power and an increase in effective sphere power. Neutralizing these prescription changes necessitates small changes to the original sphere, cylinder, and axis values, which will depend upon both the strength of the prescription and the degree of lens tilt. If the prescription has been adjusted in this manner by the free-form lens supplier, a *compensated prescription* should be provided, which represents the vertex powers—for power verification purposes—necessary to provide the wearer with the intended prescription once the lenses are in the actual position of wear.

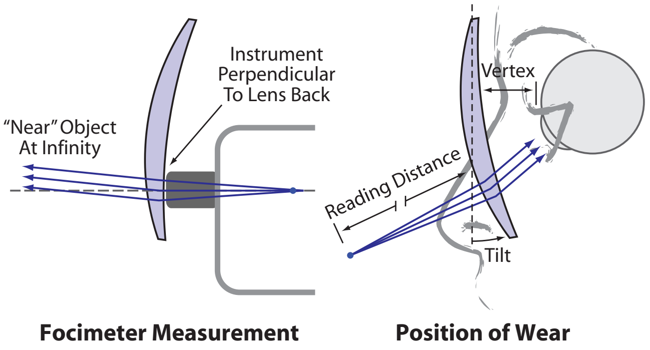


Figure 5. The optical performance of the lens as measured by a focimeter may differ significantly from the optical performance of the lens as perceived by the actual wearer with the lens in its fitted position of wear.

For a relatively thin spherical lens, the compensated sphere power S_{COMP} and cylinder power C_{COMP} required to achieve an effective sphere power S_{RX}, once the lens has been tilted by an angle θ, are given by:

$$S_{COMP} = \frac{S_{RX}}{1 + \frac{\sin^2\theta}{2n}} \qquad \text{...\textit{Compensated sphere} [1]}$$

$$C_{COMP} = -S_{COMP} \cdot \sin^2\theta \qquad \text{...\textit{Compensated cylinder} [2]}$$

where *n* is the refractive index of the lens. The compensated cylinder axis is at 180 degrees for pantoscopic tilt and at 90 degrees for face-form wrap. In the presence of prescribed cylinder power, prism, combined pantoscopic and face-form tilt, or substantial lens thickness, more complicated mathematics are necessary.[8] Further, due to the highly oblique angles of gaze utilized during near vision and to the differences in *near vision effectivity* as a result of vergence changes through curved lenses of non-negligible thickness, modifications to the prescribed near addition power may also be necessary.

In some cases, the free-form lens supplier may choose to constrain the prescription optimization at the distance and near power verification points in order to preclude the use of compensated prescriptions. Although prescription optimization will still improve the overall optics of the lens in the absence of prescription compensation, the free-form lens supplier compromises optical performance slightly in this case for the sake of simpler dispensing. The reduction in potential optical performance within the central viewing zones will depend upon the strength of the original prescription and the fitting geometry.

Position of Wear Customization

Various position of wear parameters must be assumed while ray tracing the lens design during prescription optimization, including the vertex distance, pantoscopic (vertical) tilt, face-form (horizontal) wrap, and preferred reading distance. Often, "default" values are utilized, which represent reasonable averages from the population. Nevertheless, these fitting parameters vary considerably among spectacle wearers. For instance, lens tilt ranges anywhere from zero to 20 degrees in practice. Moreover, significant differences in the position of wear or fitting geometry can have a noticeable impact upon the optical performance of the lens as perceived by the wearer, as demonstrated in Figure 6.

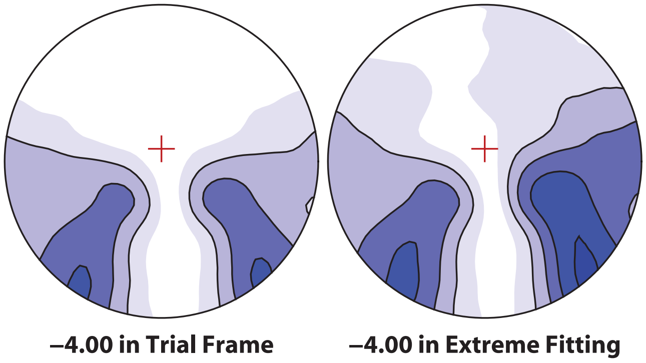
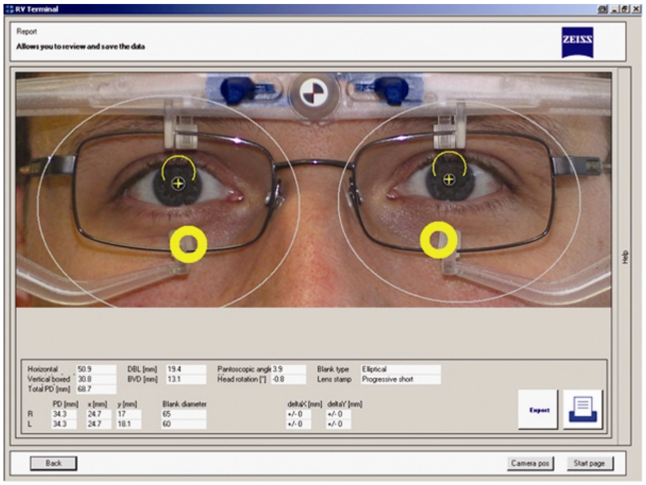


Figure 6. Extreme position-of-wear fitting geometries can have a marked effect upon optical performance compared to the fitting geometry of the trial frame—particularly in higher prescriptions—as demonstrated by adding 15 degrees of pantoscopic tilt and 10 degrees of face-form wrap to this −4.00-diopter progressive lens.

Position of wear customization relies on fine-tuning the lens design during the prescription optimization process for the wearer's actual position of wear parameters. This maximizes the optical performance of the lens design, regardless of the fitting geometry of the lens. Position of wear measurements must be supplied to the free-form surfacing laboratory, and the gain in accuracy realized during the optimization process will depend upon the number of additional position of wear measurements provided. These measurements can be taken with a variety of devices that range from inexpensive hand-held dispensing tools to extremely accurate computerized centration systems that capture these measurements automatically from digital images of the wearer (Figure 7).



Report

Allows you to review and save the data

Horizontal 50.9
Vertical boxed 30.8
Total PD [mm] 68.7
DBL [mm] 19.4
BVD [mm] 13.1
Pantoscopic angl 3.9
Head rotation [°] -0.8
Blank type Elliptical
Lens stamp Progressive short

	PD [mm]	x [mm]	y [mm]	Blank diameter	deltaX [mm]	deltaY [mm]
R	34.3	24.7	17	65	+/- 0	+/- 0
L	34.3	24.7	18.1	60	+/- 0	+/- 0

Figure 7. Dispensing tools for taking accurate position of wear measurements include highly sophisticated digital centration systems capable of capturing a variety of measurements (photo courtesy of Carl Zeiss Vision GmbH).

Advanced Forms of Customization

Prescription customization and position of wear customization fine-tune the basic progressive lens design in order to ensure consistent optical performance, regardless of the wearer's prescription requirements or fitting geometry. These forms of free-form customization simply replicate the "ideal" performance of the basic lens design. However, advanced forms of customization are also available that allow lens designers to further improve visual performance and satisfaction by significantly modifying the basic progressive lens design based on information specific to the individual wearer. These advanced forms of customization realize the full potential of free-form technology by providing the wearer with truly individualized progressive lens designs.

The "degrees of freedom" available to the progressive lens designer include, but are not necessarily limited to, the length of the progressive corridor, the relative balance between the size of the distance zone and the size of the near zone, and the relative balance between the size of the central viewing zones and the softness of the periphery. The ability to manipulate these variables in real time affords the lens designer with a multi-dimensional customization space of lens design possibilities, as illustrated in Figure 8.

With sufficiently advanced software tools capable of real-time optical design, a free-form lens supplier can generate a completely arbitrary lens design that has been fully parameterized using values specific to the wearer. Alternatively, an appropriate lens design that best matches the wearer may be selected from a range of possible lens designs, in lieu of the more complex and resource-intensive

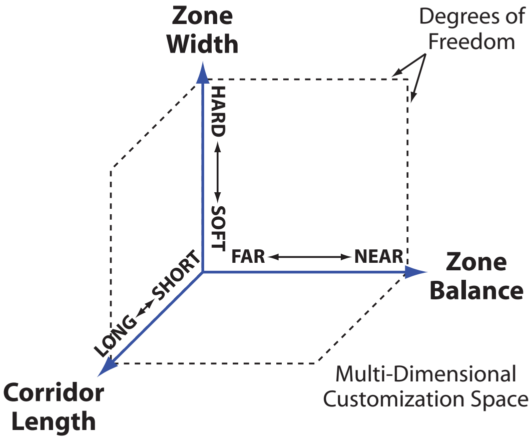


Figure 8. The degrees of freedom available for manipulating the geometry of a progressive lens design represent a multi-dimensional customization space of lens design possibilities.

process of optical design in real time. However, the customization afforded by this latter approach will be limited by the number of suitable options available in the free-form lens supplier's repository of possible lens designs, including the number of lens designs available with unique corridor lengths, unique viewing zone balances, and so on.

Of course, determining how to best manipulate these lens design parameters for a given wearer requires the application of extensive vision science and clinical research. In some cases, new dispensing technologies designed to capture critical measurements and wearer feedback may be required. Currently, advanced free-form lens designs are available that are tailored to the wearer's chosen frame style, visual demands typical of the wearer's lifestyle, and physiological behavior patterns captured from biometric measurements of the wearer.

Frame Style Customization

Most general-purpose progressive lenses are designed to work well in *conservative* frame styles. Although many modern progressives will perform adequately at 17- or 18-millimeter fitting heights, many lens designs may not achieve optimal optical performance with fitting heights below 20 to 22 millimeters. Although various "short corridor" progressive lenses are now available for shorter fitting heights, these lens designs are not without their compromises. The shorter the length of the progressive corridor, the more the optics of the lens design must be "compressed," leaving wearers to tolerate reduced intermediate utility, higher levels of peripheral blur, and narrower viewing zones, in accordance with Minkwitz's theorem.

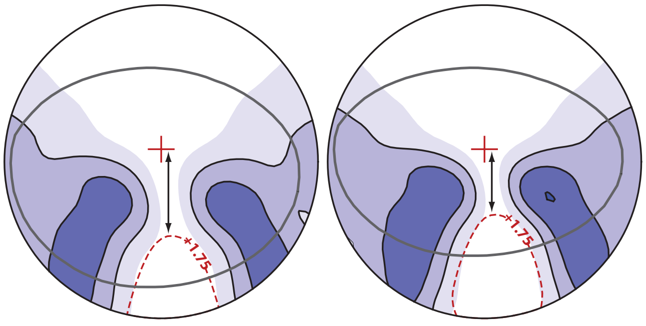
Moreover, many recent short-corridor progressive lenses have been engineered for *ultra-small* frames requiring extremely short fitting heights. Eye care professionals may be forced to choose between lens designs engineered to work well either in conservative frames or in ultra-small frames, and to determine at what fitting height to switch from one to the other. Inevitably, unless the corridor length of the chosen lens design happens to coincide with the optimal length required for a particular wearer's chosen frame style, the wearer must tolerate unnecessary optical compromises.

Frame style customization relies on matching the corridor length of the lens design to the chosen frame style, based upon the fitting height measurement and possibly other frame dimensions, in order to maximize near vision utility without unnecessarily compromising optical performance in other regions of the lens (Figure 9). This allows the optics of the lens design to take full advantage of the available lens area. Typically, this customization is based on the standard fitting height measurement supplied to the laboratory. A progressive lens design having the most suitable corridor length for the frame can then be chosen from a range of two or more corridor length options, or the corridor length of the design may be continuously varied over a range of possible values with the use of sufficiently advanced software.

In addition to customization based on fitting height or frame size, it is also possible to manipulate the optics and form of the lens based on the overall "shape" of the frame and other opto-mechanical requirements. For instance, the optics and form of the lens design can be tailored to facilitate glazing in exotic frames styles or to the use of non-standard base curves. With the increasing popularity of steeply curved and highly wrapped eyewear, which often necessitate complex atoric lens designs for optimal performance, this application of free-form technology is becoming increasingly relevant.

Lifestyle Customization

The ideal progressive lens design for a given wearer will depend in no small part upon the visual demands specific to his or her lifestyle. It has been demonstrated that preference for progressive lens designs can vary with the unique visual needs of the wearer.[9] Progressive lens wearers more frequently engaged in tasks associated with far vision will often prefer lens designs with larger distance zones, whereas wearers with greater near vision demands may prefer lens designs with larger near zones (Figure 10). Moreover, a low hyperope who only wears her spectacles while reading may prefer a larger near zone, whereas a low



"Long Corridor" Design **"Short Corridor" Design**

Figure 9. The geometry of a progressive lens design can be customized based on the size of the frame by altering the corridor length of the lens design.

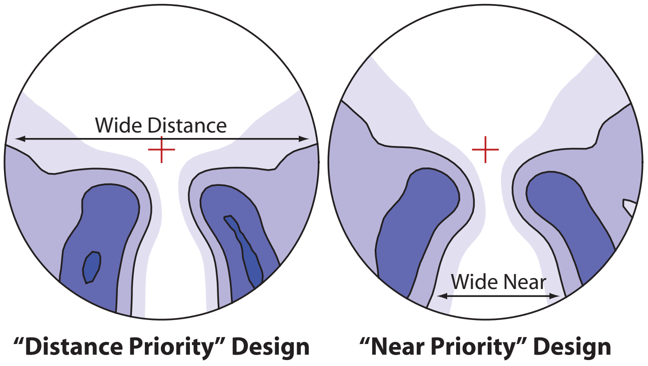


Figure 10. The geometry of a progressive lens design can be customized based on visual lifestyle requirements by altering the balance between the size of the distance viewing zone and the size of the near zone.

myope who removes her spectacles to read may prefer a larger distance zone.

Lifestyle customization relies on assessing the relative visual demands of the wearer in order to determine the ideal balance between the distance and near viewing zones of the lens design. Relevant lifestyle information may be captured using computer screening or a questionnaire of some form. A progressive lens design having the most suitable viewing zone configuration for the wearer can then be chosen from a range of possible lens designs, or the viewing zone balance of the design may be continuously varied to match the exact balance indicated for the wearer.

The relative suitability of common progressive lens designs for different viewing tasks has been previously evaluated.[10] Many of these lens designs are positioned as "general-purpose" lenses in the marketplace, suggesting that these lens designs do not intentionally differ from a viewing zone balance consistent with equal distance and near vision requirements. The range of possible viewing zone balances available commercially is therefore limited, at best. Customized progressive lenses delivered via free-form lens surfacing, however, are not constrained by the same limitations in availability. Additionally, while choosing one of these lens designs based on measurements of viewing zone size offers some degree of freedom, this relies on an accurate assessment of the optical performance of each lens, which may not be readily accessible in many cases.

Biometric Customization

It has also been demonstrated that individuals vary in their habitual head movement propensity for a given angle of gaze, especially when fixating objects at significant lateral viewing angles. The ratio of the angle of head rotation to the total angle of gaze is known as *gain*, so that gain is equal to head angle divided by gaze angle. Gain ranges from zero (for eye movement only) to 100 percent (for head movement only). Individuals who tend to exhibit habitually higher gain, or relative head movement, are frequently referred to as "head movers," whereas individuals who exhibit lower gain are referred to as "eye movers."[11,12]

For some wearers, the limited width of the viewing zones of a progressive lens may restrict lateral eye movement, necessitating an increase in head movement gain by the wearer.[13] Even when eye movement is not significantly restricted, reading efficiency may be noticeably reduced by narrower viewing zones, subsequent to an increase in gaze stabilization time and in the number of reading regressions.[14] It has been suggested that these factors contribute to the adaptation problems experienced by some progressive lens wearers.

Consequently, "eye movers" may potentially benefit from the use of progressive lens designs with wider central viewing zones. "Head movers," on the other hand, will fixate an object with a ballistic eye movement, during which vision is suppressed, while initiating a much slower compensatory head movement. During this head movement, the visual field may be disrupted by the changing prism and magnification effects across the progressive lens design as the gaze remains relatively stable. Therefore, "head movers" may benefit from designs with softer gradients of power and astigmatism that minimize image swim, skew distortion, and other optical imaging defects associated with prism and magnification gradients (Figure 11).

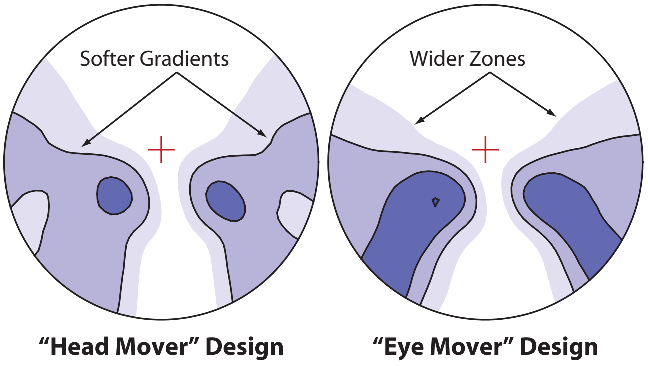


Figure 11. The geometry of a progressive lens design can be customized based on head-tracking data and other forms of biometric feedback by altering the balance between the size of the central viewing zones and the gradients of addition power and astigmatism.

Biometric customization relies on the measurement of the physiological interaction of the wearer with his or her visual environment. For biometrically customized progressive lenses, a head-tracking device or similar instrument is required. Head-tracking measurements are captured by a computer during key viewing tasks, which often involve either fixating flashes of light presented at two lateral viewing angles or performing an actual reading task (Figure 12). Again, the progressive lens design having the most suitable geometry for the wearer can be chosen from a range of possible lens designs, or the geometry of the design may be continuously varied to match the exact balance indicated for the wearer, depending upon the sophistication of the free-form supplier's software tools.

Lens Surface Configuration

With two separate surfaces to work with, the optical design and prescription components of a free-form progressive lens can be applied to the lens blank in variety of possible configurations. Each configuration represents a particular combination of factory-finished, traditionally-surfaced, and free-form-surfaced lens curves. The lens surfaces involved range in complexity from simple spherical surfaces to optimized progressive surfaces that have been combined with the prescription sphere and cylinder curves.

As described earlier, a common configuration employs a semi-finished spherical surface on the front and a free-form-surfaced progressive surface on the back that has been combined with the normal prescription curves. In this case, the actual progressive lens design is directly surfaced. An alternative approach employs a semi-finished (that is, prefabricated) progressive surface on the front and free-form-surfaced prescription curves on the back that have been optically optimized. There is also a class of "dual surface" configurations that employ a *partial* or "split" progressive surface on the front and a partial progressive surface on the back that has been combined with the prescription curves.[15]

Figure 12. For "biometrically-customized" progressive lens designs, special head-tracking devices are required (photo courtesy of Carl Zeiss Vision GmbH.)

Although it is sometimes claimed that "splitting" the progressive design between the front and back surfaces reduces unwanted astigmatism, the actual differences in performance are generally small. Because a typical spectacle lens represents an "optical system" of fairly negligible thickness, the optics of each surface are essentially additive. The optical powers across the lens can be distributed between both surfaces with very little change in effective optical performance. Consequently, the placement of the actual progressive optics, whether on the front surface, back surface, or split between both, has very little impact on the inherent unwanted astigmatism of the lens design (Figure 13).

The magnitude of astigmatism produced by a progressive lens design is not significantly influenced by the choice of surface placement. Nevertheless, there may be some minor optical benefits to the use of a back-surface progressive lens configuration. Although the *vertex power* will remain unchanged, the *equivalent power* and *magnification* across the lens will vary depending upon the surface used for the progressive optics. In particular, differences in curvature on the front surface will contribute to spectacle magnification effects. Therefore, a slight reduction in skew distortion may

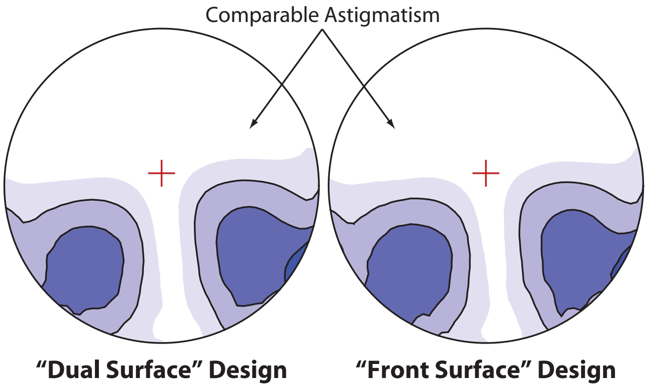


Figure 13. The ray-traced optical astigmatism for a "dual surface" progressive lens design and a conventional front-surface lens design that has been similarly optimized are virtually identical.

be obtained when the progressive optics are located on the back surface. Additionally, because the “limiting aperture” of the lens—delineated by the zones of clear vision—is brought closer to the eye, slightly wider fields of view may be obtained in some cases when the progressive optics are located on the back surface.

For free-form lens suppliers, the choice of free-form surface configuration is often influenced by many non-optical factors, such as ease of manufacturing and any limitations imposed by existing patents and similar intellectual properties. For instance, back-surface progressive lens configurations limit the number of surfaces that must be “worked,” which offers certain production advantages while eliminating the potential for misalignment between the front and back surfaces. Front-surface progressive lens configurations, on the other hand, may be available in a wider prescription range, since the rear prescription surface is not limited by the dynamic range of free-form surfacing equipment.

“Wavefront” Corrections in Spectacle Lenses

Recently, there has been a great deal of interest in “wavefront” technology as applied to both refractive surgery and spectacle lenses. Of course, this interest is primarily driven by recent advances in laser refractive surgery that allow surgeons to reduce the “high-order” aberrations of the eye, in addition to the traditional spherical and cylindrical refractive errors, using wavefront-guided ablation. The ultimate goal of wavefront-guided refractive surgery is to achieve *supernormal vision*, with better than “normal” visual acuity and contrast sensitivity, or at least to improve postoperative results compared with traditional refractive surgery.

Several spectacle lens manufacturers are now marketing lens designs that also minimize “higher-order” wavefront aberrations. These spectacle lenses generally fall into one of two categories: either spectacle lenses that are claimed to reduce the high-order aberrations of the *spectacle lens*, itself, or spectacle lenses that are claimed to reduce the high-order ocular aberrations of the *wearer’s eye*. Unfortunately, there has been a great deal of confusion in the marketplace surrounding the application of this technology to ophthalmic lens design. It is important to distinguish between the correction of the wavefront aberrations of the *eye* and the wavefront aberrations of a *spectacle lens*.

Review of Wavefront Aberrations

It is now well understood that wavefront aberrations represent one of several possible ways of characterizing the optical errors of a lens or system. At any point across the aperture of the system, such as the pupil of the eye, the wavefront error is the separation, or difference in *optical path length*, between the actual wavefront and the ideal wavefront. In the presence of uncorrected refractive errors and other optical aberrations, the actual wavefront is often flatter or steeper than necessary and distorted in shape. After the errors in height between the actual, aberrated wavefront and the ideal wavefront surface have been determined, these error measurements are typically fitted with one of several possible sets of *basis functions*. These functions allow the complex shape of the wavefront errors to be broken down, or *decomposed*, into an assortment of more basic component shapes.

One of the most common sets of basis functions used in ophthalmic optics is the *Zernike polynomial series*.[16] Each Zernike basis function, referred to as a *mode*, comprises a *radial order* component indicating the variation of the function from the center of the pupil and a *meridional frequency* component indicating the number of sinusoidal repetitions of the radial component around the pupil. Each Zernike mode is associated with a particular type of optical error, or wavefront aberration, allowing the wavefront errors to be described as a combination of quantities of more basic optical aberrations. Individual Zernike modes are commonly grouped by their radial order, which indicates the increasing dependence of the modes on pupil size:

- *Low-order* aberrations are Zernike modes of the “second order” and lower. *Second-order* aberrations include defocus and astigmatism, which are essentially equal to errors in sphere power and cylinder power, and are usually the most detrimental to the quality of vision for normal eyes. The *zeroth-order* (that is, piston) and *first-order* (that is, tilt) modes are generally neglected in measurements of image quality.
- *High-order* aberrations are the remaining Zernike modes of the “third order” and higher. High-order aberrations include coma, trefoil, spherical aberration, and so on. High-order aberrations generally have less impact on vision quality in normal eyes, and are usually not of consequence until the lower-order aberrations of

> defocus and astigmatism have been substantially ameliorated.[17]

Additionally, each Zernike mode has a coefficient associated with it indicating the quantity of that particular Zernike aberration present in the actual wavefront surface. The overall magnitude of the wavefront errors is often stated in terms of the RMS (or root-mean-square) error of the wavefront. The RMS error is essentially equal to the *standard deviation*—a statistical measure of variation—of the wavefront errors from the ideal wavefront across the reference pupil. The RMS wavefront error can be calculated directly from Zernike coefficients by taking the square-root of the sum of the squares of the coefficients.

Wavefront Aberrations in Progressives

Conventional single vision and bifocal spectacle lenses that have been properly fabricated to the intended prescription will produce no second-order Zernike aberrations *along* the optical axis of the lens. Second-order Zernike aberrations will occur, however, at *oblique* angles of view due to the introduction of the primary *Seidel* optical aberrations known as *oblique astigmatism* (producing Zernike astigmatism) and *curvature of the field* (producing Zernike defocus). These two optical aberrations are generally minimized with the use of “best form” base curves or aspheric lens designs. Conventional single vision and bifocal spectacle lenses typically produce only negligible levels of higher-order Zernike aberrations in most prescription powers, since the relatively small pupil diameter of the eye effectively “stops down” the aberrated ray bundle, thereby reducing the resulting point spread of the image. Progressive lenses, on the other hand, can produce significant levels of certain higher-order aberrations, in addition to second-order aberrations, due to the variation in refractive power and astigmatism across the progressive surface.

In progressive lens design, the *second-order* aberrations are primarily due to unwanted surface astigmatism (producing Zernike astigmatism) and excess addition or plus power for a given viewing distance (producing Zernike defocus). Moreover, because progressive lens surfaces utilize continuously changing curvatures, which are associated with the second derivatives of the surface, in order to produce a progressive change in addition power, this class of surfaces has non-zero *third* derivatives. Consequently, progressive lenses produce certain levels of the higher-order wavefront aberrations associated with the third derivatives of a surface, specifically, the *third-order* Zernike aberrations known as *coma* and *trefoil*.

The presence of coma across a progressive lens surface can be deduced from Figure 14. Classic coma is due to an asymmetric variation in refractive power and magnification across the lens for off-axis object points. The change in refractive power across a progressive lens surface produces a very similar effect. As the line of sight passes down the progressive corridor of the lens, the power at the upper margin of the pupil differs from the power at the lower margin by an amount roughly equal to the product of the pupil diameter and the rate of change in addition power at that particular location. In fact, coma is directly proportional to the rate of change in mean addition power.

Some additional insight into the nature of wavefront aberrations in progressive lenses may be deduced by comparing the shape of a progressive lens surface directly to the actual basis functions used to “build” a given wavefront (Figure 15). The Zernike basis functions used to represent the contribution of vertical coma (Z_3^{-1}) and oblique trefoil (Z_3^{-3}) to the overall shape of a wavefront surface are given by the following functions in Cartesian form:[18]

$$Z_3^{-1} = N\left(3yx^2 + 3y^3 - 2y\right) \qquad \ldots\textit{Vertical coma} \text{ [3]}$$

$$Z_3^{-3} = N\left(3yx^2 - y^3\right) \qquad \ldots\textit{Oblique trefoil} \text{ [4]}$$

where N is a normalizing term equal to √8 for the third-order Zernike aberrations. Neglecting the linear ($2y$) term from the coma function, since this term represents only the contribution of lower-order tilt or prism, the sum of these two basis functions is given by:

$$f_{ZZ}(x, y) = Z_3^{-1} + Z_3^{-3} = 2\sqrt{8}\left(y^3 + 3yx^2\right) \qquad \text{[5]}$$

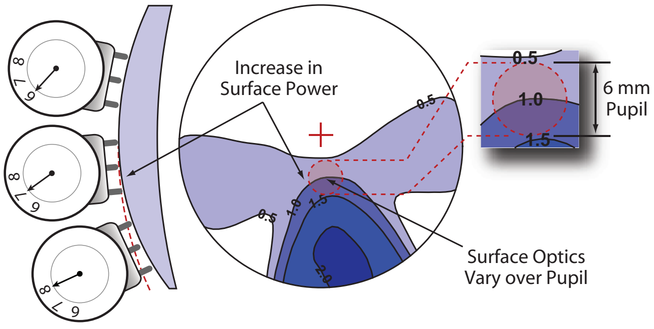


Figure 14. The progression of addition power across a progressive lens surface causes the power to vary over the finite diameter of the wearer’s pupil, introducing a coma-like wavefront aberration.

This equation is identical in form to that of the surface height function *z* of the simple "elephant trunk" progressive lens model described in the companion paper, namely:[19]

$$z(x,y)=\frac{g}{6}\left(y^3+3yx^2\right) \qquad \text{...}\textit{Elephant trunk surface}\ [6]$$

where *g* is related to the *power law* (δ*Add*), or rate of change in addition power, of the lens design as follows:

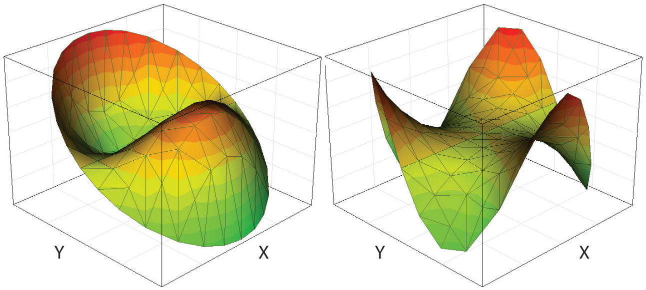
$$g=\frac{\delta Add}{1000(n-1)} \qquad [7]$$

Consequently, this simple progressive lens surface is similar in shape to a combination of Zernike vertical coma and Zernike oblique trefoil. The progression of addition power across the lens surface essentially acts as a coma-like wavefront aberration over the entire "aperture" of the progressive zone, while the astigmatism-free umbilic is the result of a trefoil-like wavefront aberration over the same region. The magnitude of these two Zernike modes depends on the addition and corridor length of the lens.

An analytical model has been described for computing the third-order wavefront aberrations of the elephant trunk surface.[20] This analytical model can be derived with the aid of some basic algebraic manipulation. Zernike basis functions are calculated over a unit circle. Therefore, the *x* and *y* terms of these basis functions must first be "normalized" by dividing each by the maximum pupil radius ρ:

$$f_{ZZ}(x,y)=2\sqrt{8}\left[\left(\frac{y}{\rho}\right)^3+3\frac{y}{\rho}\left(\frac{x}{\rho}\right)^2\right]$$

$$f_{ZZ}(x,y)=\frac{2\sqrt{8}}{\rho^3}\left(y^3+3yx^2\right) \qquad [8]$$



Zernike Coma (Z_3^{-1}) **Zernike Trefoil (Z_3^{-3})**

Figure 15. The action of a simple progressive lens surface can be described by a combination of vertical coma and oblique trefoil wavefront aberrations.

Next, the approximate refractive power of the elephant trunk surface *z* is expressed in terms of a wavefront profile function *w*, in microns (μm or mm^{-3}), using:

$$w(x,y)=z(x,y)\cdot 1000(n-1)$$

$$w(x,y)=\frac{g}{6}\left(y^3+3yx^2\right)\cdot 1000(n-1) \qquad [9]$$

however, since *g* is given by Equation 7:

$$g=\frac{\delta Add}{1000(n-1)}$$

the wavefront profile function simplifies to:

$$w(x,y)=\frac{\delta Add}{6}\left(y^3+3yx^2\right) \qquad \text{...}\textit{Wavefront profile}\ [10]$$

The combined Zernike basis functions are then equated to the wavefront profile function of the elephant trunk surface:

$$C_3\frac{2\sqrt{8}}{\rho^3}\left(y^3+3yx^2\right)=\frac{\delta Add}{6}\left(y^3+3yx^2\right) \qquad [11]$$

where C_3 is the Zernike coefficient of the combined third-order coma and trefoil functions. This coefficient essentially represents the amount by which to "scale" the coma and trefoil functions in order to produce the desired wavefront profile of the elephant trunk progressive surface. It is also equal to the RMS wavefront error of the Zernike modes. Canceling like terms and solving for the Zernike coefficient C_3 yields:

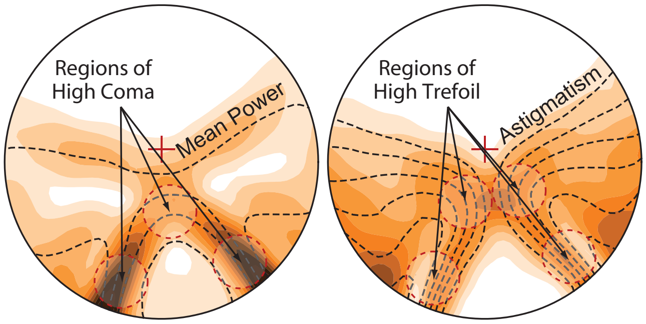
$$C_3=\frac{\delta Add}{12\sqrt{8}}\rho^3 \qquad \text{...}\textit{Third-order coefficient}\ [12]$$

This demonstrates that the simple progressive lens model presented earlier produces equal amounts of third-order coma and trefoil wavefront aberrations, which are constant over the progressive region of the surface. This is to be expected, since this simple progressive lens surface has constant third derivatives. Moreover, these aberrations are proportional to the rate of change in addition power (δ*Add*) along the umbilic of the lens surface. Additionally, this equation for the Zernike coefficient demonstrates the pupil size dependence (ρ^3) of the third-order wavefront aberrations produced by a progressive lens.

Unlike the simple progressive lens model presented earlier, modern progressive lenses employ non-circular cross-sections and a power law that varies non-linearly along the corridor. Nevertheless, the third-order aberrations in these lenses will vary with the rate of change in surface power at any point. Coma and trefoil will be highest in regions wherein the addition power and surface astigmatism are changing most rapidly (Figure 16). Moreover, since high-order aberrations are dependent on the rate of change in surface power, these aberrations will be more significant in progressive lenses with shorter corridor lengths or higher addition powers, in accordance with Minkwitz's theorem.

Recently, progressive lens designers have begun paying closer attention to high-order aberrations, and in some cases even patenting progressive lens designs with reduced high-order aberrations, such as coma.[21] Unfortunately, you cannot *eliminate* the high-order aberrations produced by a progressive lens surface, just as you cannot eliminate unwanted surface astigmatism. In fact, for modern progressive lens designs at least, average levels of high-order aberrations calculated globally over the central lens surface are fairly similar in magnitude.[22] Nevertheless, you *can* judiciously *manage* both the low- and high-order aberrations in a progressive lens.

Just as there are two general approaches to the management of second-order astigmatism, by either spreading it out to "soften" the design or confining it to smaller regions to "harden" the design, there are also two intimately related approaches to the management of third-order aberrations. A "soft" lens design with gradual power changes will frequently yield relatively low levels of high-order aberrations over the entire lens, whereas a "hard" lens design with rapid power changes will yield lower levels of high-order aberrations in the central distance and near viewing zones while creating greater levels at the viewing zone boundaries and within the progressive corridor. In general, for a given corridor length and addition power, minimizing high-order aberrations in any particular region will be at the expense of inducing higher levels of high-order aberrations elsewhere.



Coma and Mean Add Power **Trefoil and Astigmatism**

Figure 16. High-order aberrations in a progressive lens are greatest in regions where the addition power and astigmatism are changing most rapidly—particularly in the vicinity of the central viewing zones and within the progressive corridor.

Although higher-order aberrations may result in a reduction in image quality and a loss of contrast, low-order aberrations generally account for the greatest impact on vision quality in progressive lenses. In particular, unwanted astigmatism dominates much of the lens surface. Further, in contrast to the case of low-order aberrations, clinical research has demonstrated that the high-order aberrations in progressive lenses are seldom any greater in magnitude than the inherent high-order aberrations of a typical wearer's eyes.[23]

Research has also demonstrated that the impact of high-order aberrations on visual acuity in the progressive corridor, where these aberrations are often highest, are negligible. Additionally, the caustic focus produced in the presence of small amounts of high-order aberrations may possibly improve the wearer's depth of focus and tolerance to the blur caused by the second-order aberrations of the lens. In fact, aberration coupling between the high-order aberrations of the progressive lens and the high-order aberrations of the eye can sometimes yield better visual acuity than obtained with the naked eye.[24] Nevertheless, the high-order aberrations produced by a progressive lens will have at least some impact on the wearer's vision, and therefore represent a meaningful quantity to evaluate during the optical design process.

Spectacle Correction of Ocular Aberrations

It should be emphasized that, in general, minimizing the high-order wavefront aberrations produced by a *spectacle lens* will *not* provide the wearer with better than his or her best corrected visual acuity. The high-order aberrations of the *eye* can only be reduced after first measuring the eye with a wavefront sensor, such as an *aberrometer*, and then precisely customizing an optical component based on those measurements.[25] The technical limitations involved, however, in the spectacle correction of high-order ocular aberrations make the application of this type of technology challenging, if not prohibitive.

Spectacle lenses cannot eliminate high-order aberrations over a wide field of view without introducing additional, lower-order wavefront aberrations as the eye rotates from the center of any "ideal" wavefront correction that has eliminated high-order aberrations. For instance, correcting second-order aberrations results in induced first-order prism as the eye moves from the center of the correction (that is, *Prentice's rule*), while correcting third-order aberrations results in induced second-order astigmatism and defocus errors. This effect can be appreciated to some extent by adding a horizontal or vertical offset to the terms of a Zernike basis function, and then expanding the new binomials, so that a term such as $2x^2$ becomes $2(x + \Delta x)^2 = 2x^2 + 4x\Delta x + 2\Delta x^2$ in the presence of a horizontal offset (Δx), now with lower-order $4x\Delta x$ and $2\Delta x^2$ terms.

In fact, just a few millimeters of movement of the line of sight from the center of an ideal wavefront correction will introduce new, lower-order wavefront errors that are actually greater in magnitude than the higher-order aberrations initially eliminated, and these errors will progressively worsen with increasing movement.[26] Since the human eye remains in a constant state of movement, correcting the high-order aberrations of the eye with a spectacle lens may frequently result in poorer vision quality than leaving the high-order aberrations uncorrected. Thus, the sensitivity of high-order aberrations to alignment errors during normal gaze changes places drastic limits on the potential benefits derived from correcting these aberrations with a spectacle lens. Moreover, for the correction of presbyopia with progressive lenses, which suffer from significant second-order aberrations, the gross optical performance of the progressive lens design will undoubtedly serve as a greater indicator of wearer satisfaction.

Although there are significant optical limitations associated with eliminating the high-order aberrations of the eye with a traditional spectacle lens, it may be possible to determine better second-order spectacle corrections by at least considering the effects of these aberrations. Conventional autorefractors have not replaced subjective refraction as the best method to determine the final second-order spectacle prescription for a given individual, and even subjective refraction suffers from variability and limits in measurement precision. The optimum prescription is influenced by not only the second-order refractive errors, but also by the higher-order aberrations of the eye and the neural processing of the visual system. Determining the endpoint of refraction by taking into account the effects of high-order aberrations on power and blur, as well as the neural processing of the visual system, may result in more accurate and repeatable second-order prescriptions.[27]

Future of Progressive Lens Design

In the one hundred years since progressive lenses were first described, the design and manufacture of these lenses has ebbed and flowed between enabling technologies and the lens designs that they can produce. Each advance in manufacturing technology has facilitated further advances in lens design. In particular, the introduction of numerically-controlled cutting and high-speed computing set the stage for a rapid expansion of progressive lens production and design innovation toward the end of the 1970s. By the late 1980s, continued research and development between competing lens manufacturers had yielded significantly improved progressive lens designs that quickly became the preferred choice of vision correction for presbyopia.

Incremental advancements in progressive lens design have continued through ongoing vision research. Over the last decade, however, the most significant trend in progressive lens design has been the emergence of free-form manufacturing technologies that facilitate the design and production of progressive lens surfaces in real time. This technology makes possible the application of various forms of lens design customization for the individual wearer. Free-form lens surfacing has also allowed progressive lens manufacturing to shift partially from mass production at centralized manufacturing sites to on-demand production at local prescription laboratories.

The customization and sophistication of lens designs will play an increasing role in free-form technology as lens suppliers attempt to differentiate their products by offering more wearer benefits—benefits that can only be realized through this type of manufacturing platform. Moreover, since additional input data are often required in order to implement many forms of customization, dispensing technologies may play an increasingly important role as well. Currently, the sophistication of free-form lens designs varies considerably from lens supplier to lens supplier. Some free-form lens suppliers offer lenses that are virtually identical in performance to comparable mass-produced lenses, whereas other free-form lens suppliers offer highly customized progressive lenses that are parameterized for the individual wearer. It seems likely that these two paths—low-cost manufacturing and specialized customization—will continue to be developed in parallel.

Additionally, interest in the application of wavefront technology to spectacle lenses has continued to increase.

While the optical limitations involved may preclude correcting the high-order aberrations of the eye with a spectacle lens, lens designers will continue to explore the impact of optical design on the high-order aberrations of the actual spectacle lens. Furthermore, with increasing numbers of aberrometers appearing in practices, the drive to determine more accurate second-order refractions by considering the effects of higher-order ocular aberrations may also become more widespread.

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